GJ

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes

Final Examination, Fall 2007

Time: Noon - 3.00 pm **Thursday, December 13, 2007**

Instructions: **Attempt Questions 1 and 2, and one of 3 and 4.**
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Problem #1 (40 points)

Stacks of a type of fuel cells can be modelled as two parallel flat plates between which gasses are passed. This is to be analyzed.

A gas flows at a steady rate between two parallel, horizontal plates separated by a gap of thickness **2δ**. Assume the width of the plates, in the direction normal to the flow direction, is large. The flow is laminar and fully developed, and heat at a constant flux $\mathbf{q_W}$ enters into the gas from both walls. Assume the fluid properties - density (ρ), heat capacity ($\mathbf{C_p}$), thermal conductivity (**k**) and thus thermal diffusivity (α) are constants.

Develop an expression for the Nusselt number (**Nu**) for the system in the region where the temperature profile is fully developed. Show all your steps.

Problem #2 (40 points)

Discussions and evaluations are now going on as to whether nuclear energy should be used to power the extraction of bitumen from tar sands. This problem relates to a pilot nuclear reactor.

A fuel rod of enriched uranium oxide is shaped as a long bar, 6cm by 4cm in cross-section. Cooling water at 7°C is allowed to flow over the rod within which heat is being generated at a rate of 38.2 MW/m^3. It is important to estimate the critical heat transfer coefficient $\boldsymbol{h_c}$ around the rod, assumed constant all over the surface, for which the temperature at any point within the rod will equal the melting point of uranium oxide. The coefficient must always be greater than this value to avoid melt down of the fuel.

Use the ***integral method,*** and assume that the rod is infinitely long, to:

a) estimate $\boldsymbol{h_c}$.
b) What would the maximum temperature in the rod be if h approached infinity with the same conditions?

Show all your derivations.

Data:
Properties of uranium oxide: k = 23.1 W/m K, ρ = 10,960 kg/m^3, c_p = 240 J/kg K, Melting point 2850°C.

Problem #3 (20 points)

An underwater pipeline carrying light crude oil ruptured underneath a large but calm lake at a resort. A large volume of oil was released quickly before the line could be shut off. The oil floated to the surface and started to spread. This oil is to be contained by a boom and "vacuumed" up before it kills wildlife and damages the shores. It is to be assumed that components of the oil do not vaporize or dissolve in the water, i.e. the volume is conserved. It is also assumed there are no currents in the lake and no winds to aid the spread. The properties of the oil are also constant. The initial thickness of the oil layer on the water surface (at the start of the spread) is estimated at 1.4 cm.

A small plane was sent up after the spill to take pictures of the spreading oil slick at intervals. A total of five (5) pictures were taken and the diameters of the oil disc at the various times from the instant of pipe rupture are as follows:

Time, mins	15	177.7	365	1440	2440
Diameter, m	29.7	73.7	89.1	179.5	265.1

a) Estimate the volume of oil released from the pipe.
b) What is the minimum length of boom that should be brought to the site after 60 hours?
c) At what instants would one notice regime changes?

Problem #4 (20 points)

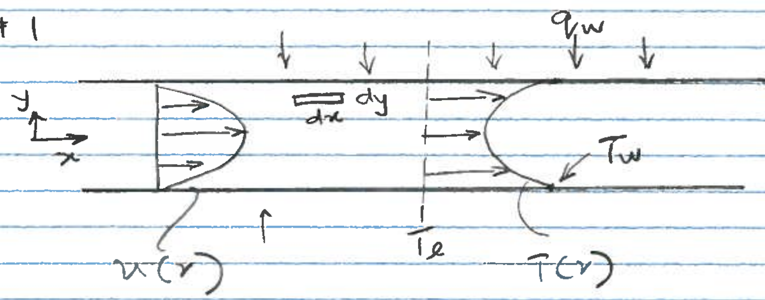
Gasoline is a complex mixture of 500 or more hydrocarbon compounds that may contain 3 to 12 carbon atoms in each molecule. The major components are alkanes which are chemically stable. Octane rating of the gasoline is based on comparing its combustion characteristics to isooctane (100, engine does not knock) and heptane (0, engine knocks badly). Unsaturated hydrocarbons in the gasoline, alkenes, alkynes and arenes, are unstable and tend to react with oxygen, produce free-radicals and polymerize to form gums that precipitate and foul engine parts. Compounds such as 1,3-cyclopentadiene are particularly implicated for gum formation. Gasoline thus degrades or "varnishes" with time and it should be used within 3 months of production. The formation of gum is to be analyzed.

Liquid cyclopentadiene maintained at T°C is poured to fill a long test tube to the brim. Air containing 21 mole % oxygen flows over the liquid surface at a slow rate and the mass transfer coefficient at the liquid-air interface is given as $\mathbf{k_C}$. Oxygen first dissolves at the boundary before diffusing into the liquid and the equilibrium constant is $\mathbf{K_A}$ ($= y_A/x_A$). The liquid initially contained no oxygen but it is saturated with nitrogen. Because the solubility of oxygen in the liquid is low, the liquid concentration is assumed constant at **c** and you may neglect the convective term of the flux. Assume the reaction between oxygen and cyclopentadiene is first order with the reaction rate constant equal $\mathbf{k_1}$, and the diffusion coefficient for oxygen diffusing in the liquid is $\mathbf{D_{AB}}$. Also assume that the liquid is not volatile.

a) Use the **integral method** to formulate the problem to obtain the concentration profiles. Show all your steps.
b) Repeat your analysis using the **differential method**. Derive the equation and state the conditions needed.

ENCH 501 Final Examination Solution Dec. 13, 2007

#1



The system is as shown in the sketch.

Choose coordinates (x, y)

Consider a differential element $dx \times dy \times 1$ (z-dir)

A force balance on the element is,

$$(\tau \cdot dx)\Big|_y - (\tau \cdot dx)\Big|_{y+dy} + P_x\, dy - P_{x+dx}\, dy = 0$$

or $$-\frac{d\tau}{dy} \cdot dy\, dx - \frac{dP}{dx}\, dx\, dy = 0$$

Since the gas is Newtonian, $\tau = -\mu \frac{du}{dy}$

Substitute and integrate

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx} \Longrightarrow \frac{du}{dy} = \left[\frac{1}{\mu}\frac{dP}{dx}\right] y + C_1$$

$$\Rightarrow \quad u = \frac{1}{2\mu}\frac{dP}{dx} y^2 + C_1 y + C_2$$

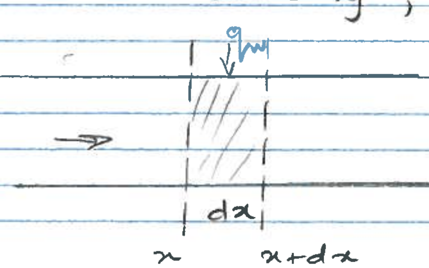
Apply b.c.

$y = 0 \quad \frac{du}{dy} = 0$ (symmetry) $\Rightarrow C_1 = 0$

$y = \delta \quad u = 0$

$$u = -\frac{dP}{dx}\frac{\delta^2}{2\mu}\left(1 - y^2/\delta^2\right) = u_{max}\left(1 - \frac{y^2}{\delta^2}\right) \qquad (1)$$

and av. velocity, $\bar{u} = \frac{2}{3} u_{max}$.



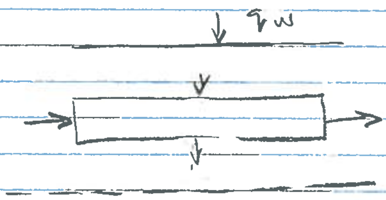
Consider a differential segment, dx

Heat gain by fluid from x to $x+dx$ is (per unit width)

$$dQ_x = q_w(2\,dx) = \dot{m}\,C_p\,dT_{bm} = \bar{u}(2\delta)\rho C_p\,dT_{bm}$$

or $$\frac{dT_{bm}}{dx} = \frac{2q_w}{\bar{u}(2\delta)\rho C_p} = \text{constant.} \qquad (2)$$

☐ Energy balance on differential element, $dx\,dy$



$$\text{Input} + \underbrace{\text{Gen}}_{0} = \text{Output} + \underbrace{\text{Accum}}_{0}$$

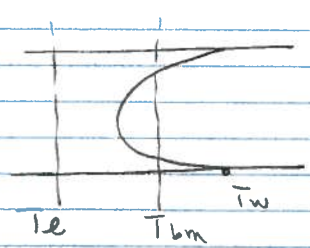
$$\left\{(u\,dy)\rho C_p T\right\}_x + (q\,dx)_{y+dy} = \left\{u\,dy\,\rho C_p T\right\}_{x+dx} + (q_y\,dx)\Big|_y$$

or $$\frac{dq_y}{dy}\,\cancel{dy}\,\cancel{dx} = u\rho C_p \frac{dT}{dx}\,\cancel{dx}\,\cancel{dy}$$

and $$q_y = +k\frac{dT}{dy} \qquad \left(\frac{dT}{dy} > 0\right)$$

$$\therefore \quad \frac{\partial^2 T}{\partial y^2} = \frac{u(y)}{\alpha}\frac{\partial T}{\partial x} \qquad (3) \quad \text{Energy Balance Equation}$$

☐



The condition of fully developed temperature profile is

$$\frac{T_w - T}{T_w - T_{bm}} \text{ is } \sim\sim\text{invariant with } x\sim\sim$$

or $$\frac{d}{dx}\left(\frac{T_w - T}{T_w - T_{bm}}\right) = 0 \quad \text{where } T_w,\ T \text{ and } T_{bm} \text{ are functions of } x$$

or $$\frac{dT}{dx} = \frac{dT_w}{dx} - \frac{T_w - T}{T_w - T_{bm}}\frac{dT_w}{dx} + \frac{T_w - T}{T_w - T_{bm}}\frac{dT_{bm}}{dx}$$

At the wall, $q_w = + k \frac{dT}{dy}\Big|_\delta$ ~~$= h(T_w - T_{bm})$~~

or $\frac{h}{k} = \frac{\partial T/\partial y|_\delta}{T_w \; T_{bm}} = -\frac{\partial}{\partial y}\left(\frac{T_w - T}{T_w - T_{bm}}\right)\Big|_\delta = \text{const}$ (4)

$\therefore$ $h/k = \text{const}$ and $T_w - T_{bm} = \text{const}$

or $\frac{dT_w}{dx} = \frac{dT_{bm}}{dx} = \frac{dT}{dx}$

Eq. (3) hence simplifies to an o.d.e.

$$\frac{d^2T}{dy^2} = \left(\frac{U_{max}}{\alpha}\frac{dT_{bm}}{dx}\right)\left(1 - y^2/\delta^2\right) = \beta\left(1 - y^2/\delta^2\right)$$

Integrate

$$\frac{dT}{dy} = \beta\left(y - \frac{1}{3}\frac{y^3}{\delta^2}\right) + c_1$$

and use symmetry condition that $y=0$, $\frac{dT}{dy} = 0$

$\Rightarrow$ $c_1 = 0$

Integrate again

$$T = \beta\left(\frac{1}{2}y^2 - \frac{1}{12}\frac{y^4}{\delta^2}\right) + c_2 \qquad (5)$$

where $c_2 = T_0$ or temperature along centre ($y=0$)

From eq. (5) obtain T_{bm} + T_w + subst into (4)

$$T_{bm} = \frac{\int_0^\delta uT\,dy}{\int_0^\delta u\,dy} = \frac{\int_0^\delta U_{max}\,\beta\left(1 - y^2/\delta^2\right)\left(\frac{1}{2}y^2 - \frac{1}{12}\frac{y^4}{\delta^2}\right)dy}{\int_0^\delta U_{max}\left(1 - y^2/\delta^2\right)dy} + T_0$$

$$T_{bm} = \frac{\beta \frac{\delta^3}{2} \int_0^1 (1-\eta^2)(\eta^2 - \frac{1}{6}\eta^4)\,d\eta}{\delta \int_0^1 (1-\eta^2)\,d\eta} + T_0$$

$$= \beta \delta^2 (13/210) \times \frac{3}{2} + T_0 = \beta \delta^2 \left(\frac{13}{140}\right) + T_0$$

$$T_w = \beta \delta^2 (5/12) + T_0$$

$$\left.\frac{dT}{dy}\right|_\delta = \beta \delta \left.\left(\frac{y}{\delta} - \frac{1}{3}\frac{y^3}{\delta^3}\right)\right|_\delta = \beta \delta (2/3)$$

$$\therefore \quad \frac{h}{k} = \frac{\beta \delta (2/3)}{\beta \delta^2 (5/12 - \frac{13}{140})} = \frac{35}{17}\frac{1}{\delta}$$

$$\therefore \quad \frac{h\delta}{k} = \frac{35}{17}$$

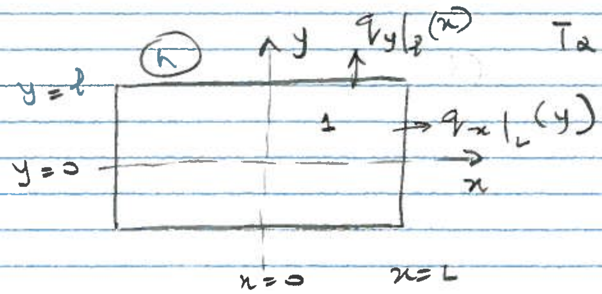
$$= 2.0588$$

$$Nu = \frac{h D_H}{k} \; ; \quad D_H = \text{hydraulic diam.} = 4 \cdot \frac{\text{flow area}}{\text{perimeter}}$$

$$= 4\delta$$

$$\therefore \quad Nu = 8.235$$

Problem # 2



The system is 2-dimensional and steady state.

Heat is generated at a rate g^+ per unit volume.

Consider quadrant 1. Perform an energy balance

$$\int_0^L q_y\Big|_\ell(x)\,dx + \int_0^\ell q_x\Big|_L(y)\,dy = g^+ L\ell \qquad (1)$$

per unit length of bar.

Since $q_x = -k\dfrac{dT}{dx}$ and $q_y = -k\dfrac{dT}{dy}$, eq. (1) becomes

$$\int_0^L \frac{\partial T}{\partial y}\Big|_\ell\,dx + \int_0^\ell \frac{\partial T}{\partial x}\Big|_L\,dy + \frac{g^+ L\ell}{k} = 0 \qquad (2)$$

A function is needed for $T(x,y)$.

Conditions:

$x=0 \qquad \dfrac{\partial T}{\partial x}=0$ symmetry

$y=0 \qquad \dfrac{\partial T}{\partial y}=0$ ✓

$x=L \qquad -k\dfrac{\partial T}{\partial x}\Big|_L = h\left(T(L,y)-T_a\right)$ (3) convective

$y=\ell \qquad -k\dfrac{\partial T}{\partial y}\Big|_\ell = h\left(T(x,\ell)-T_a\right)$

Integral Method:

Assume a temperature profile

$$T(x,y) - T_a = a + bx^2 + cy^2 + dx^2y^2 \qquad (4)$$

which is symmetric around the axes and an even function.

$$\frac{\partial T}{\partial x} = 2bx + 2dxy^2 \quad \left(\text{satisfies } x=0,\ \frac{\partial T}{\partial x}=0\right)$$

$$\frac{\partial T}{\partial y} = 2cy + 2dx^2y \quad \left(\text{satisfies } y=0,\ \frac{\partial T}{\partial y}=0\right)$$

$$T(L,y) - T_\alpha = a + bL^2 + cy^2 + dL^2y^2 = f(y) \qquad (5)$$

$$T(x,\ell) - T_\alpha = a + bx^2 + c\ell^2 + dx^2\ell^2 = f(x) \qquad (6)$$

At $x = L$

$$\left.\frac{dT}{dx}\right|_{x=L} = 2bL + 2dLy^2 = -\frac{h}{k}\left(T(L,y) - T_\alpha\right) \qquad (7)$$

from b.c.

and $$\left.\frac{dT}{dy}\right|_{y=\ell} = 2c\ell + 2dx^2\ell = -\frac{h}{k}\left(T(x,\ell) - T_\alpha\right) \qquad (8)$$

from equations (5) + (7)

$$T(L,y) - T_\alpha = (a + bL^2) + (c + dL^2)y^2 = -\frac{2kL}{h}(b + dy^2)$$

Equate the coefficients for y

$$a + bL^2 = -\frac{2kL}{h}\cdot b \quad \text{or} \quad a = -\left[\frac{2kL}{h} + L^2\right]b$$

and $$c + dL^2 = -\frac{2kL}{h}\cdot d \quad \text{or} \quad c = -\left[\frac{2kL}{h} + L^2\right]d$$

or $$a = \alpha b \quad \text{and} \quad c = \alpha d \quad ; \quad \alpha = -\left[\frac{2kL}{h} + L^2\right] \qquad (9)$$

similarly for equations (6) and (8)

$$T(x,l) - T_\alpha = (a + cl^2) + (b + dl^2)x^2 = - \frac{2kl}{h}(c + dx^2)$$

$$a = \beta c \quad \text{and} \quad b = \beta d \quad ; \quad \beta = -\left[\frac{2kl}{h} + l^2\right] \quad (10)$$

Combine equations (9) and (10)

$$a = \alpha\beta d$$
$$b = \beta d \quad (11)$$
$$c = \alpha d$$

Substitute (11) into (4)

$$T(x,y) - T_\alpha = (\alpha\beta + \beta x^2 + \alpha y^2 + x^2y^2)d \quad (12)$$

Substitute (12) into integral energy equation (2) to evaluate the unknown d.

The energy equation, using expressions above eq. (5), is

$$\int_0^L (2cl + 2dlx^2)\,dx + \int_0^l (2bL + 2dLy^2)\,dy + \frac{g^+ Ll}{k} = 0 \quad (13)$$

Integrate and simplify (13) to obtain

$$d = \frac{g^+}{2k}\left[\frac{2k}{h}(L + l) + \frac{2}{3}(L^2 + l^2)\right]^{-1} \quad (14)$$

Now substitute values

$$d = \frac{38.2\,(10^6)}{2\,(23.1)}\left[\frac{2\,(23.1)}{h}(3+2)(10^{-2}) + \frac{2}{3}(9+4)(10^{-4})\right]^{-1}$$

$$\alpha = -\left[\frac{2(23.1)\,3\,(10^{-2})}{h} + 9\,(10^{-4})\right]$$

$$\beta = -\left[\frac{2(23.1)\,2\,(10^{-2})}{h} + 4\,(10^{-4})\right] \qquad (15)$$

In the bar, the highest temperature is expected at the centre, i.e. at $x = 0$, $y = 0$.

If $T(0,0) = 2850\,°C$ and $T_\infty = 7\,°C$

Then from equation (12)

$$2850 - 7 = \alpha\beta d = \frac{\left[\frac{1.386}{h_c} + 9\,(10^{-4})\right] \times \left[\frac{0.924}{h_c} + 4\,(10^{-4})\right] \times 8.2684\,(10^{5})}{\left[\frac{2.31}{h_c} + 8.667\,(10^{-4})\right]}$$

or

$$3.4384\,(10^{-3}) = \frac{\left[\frac{1.386}{h_c} + 9\,(10^{-4})\right]\left[\frac{0.924}{h_c} + 4\,(10^{-4})\right]}{\left[\frac{2.31}{h_c} + 8.667\,(10^{-4})\right]}$$

The critical value, $h_c \doteq 182.08$ or $-2{,}684.6\ W/m^2K$

Obviously the negative value is not acceptable.

→

(b) When $h \to \infty$, from eqs. (9) and (10)

$$\alpha = -L^2 \quad \text{and} \quad \beta = -\ell^2$$

$$\therefore \quad T(x,y) - T_\infty = (L^2 - x^2)(\ell^2 - y^2)\, d$$

where $$d = \frac{g^+}{2k}\left[\frac{2}{3}(L^2 + \ell^2)\right]^{-1} \quad \text{from (14)}$$

$$= 8.2684\,(10^5)\left[\frac{2}{3}(9+4)(10^{-4})\right]^{-1}$$

$$= 9.5405\,(10^8)$$

Then

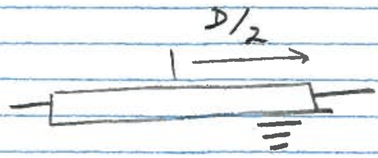
$$T(0,0) - T_\infty = (9)(4)(10^{-8})(9.5405)(10^8)$$

$$= 343.46 \ {}^\circ C$$

or $$T(0,0) = 350.46 \ {}^\circ C$$ $\longrightarrow$

Problem #3

This is the problem for oil spreading on water.
There are 3 regime -



Short time $D \propto t^{1/2}$ or $\ln D = \frac{1}{2}\ln t + c_1$

Intermediate $D \propto t^{1/4}$ or $\ln D = \frac{1}{4}\ln t + C_2$

Long time $D \propto t^{3/4}$ or $\ln D = \frac{3}{4}\ln t + C_3$

Modify data provided

D, m	ln D	t, mins	ln t
29.7	3.391	15	2.708
73.7	4.30	177.7	5.18
89.1	4.49	365	5.9
179.5	5.19	1440	7.272
265.1	5.58	2440	7.8

A plot of the logs of the data, with lines of slopes ½, ¼ and ¾ are shown.

(a) when $t = 1\,min$, $\ln t = 0$. This is close to the start of the release.

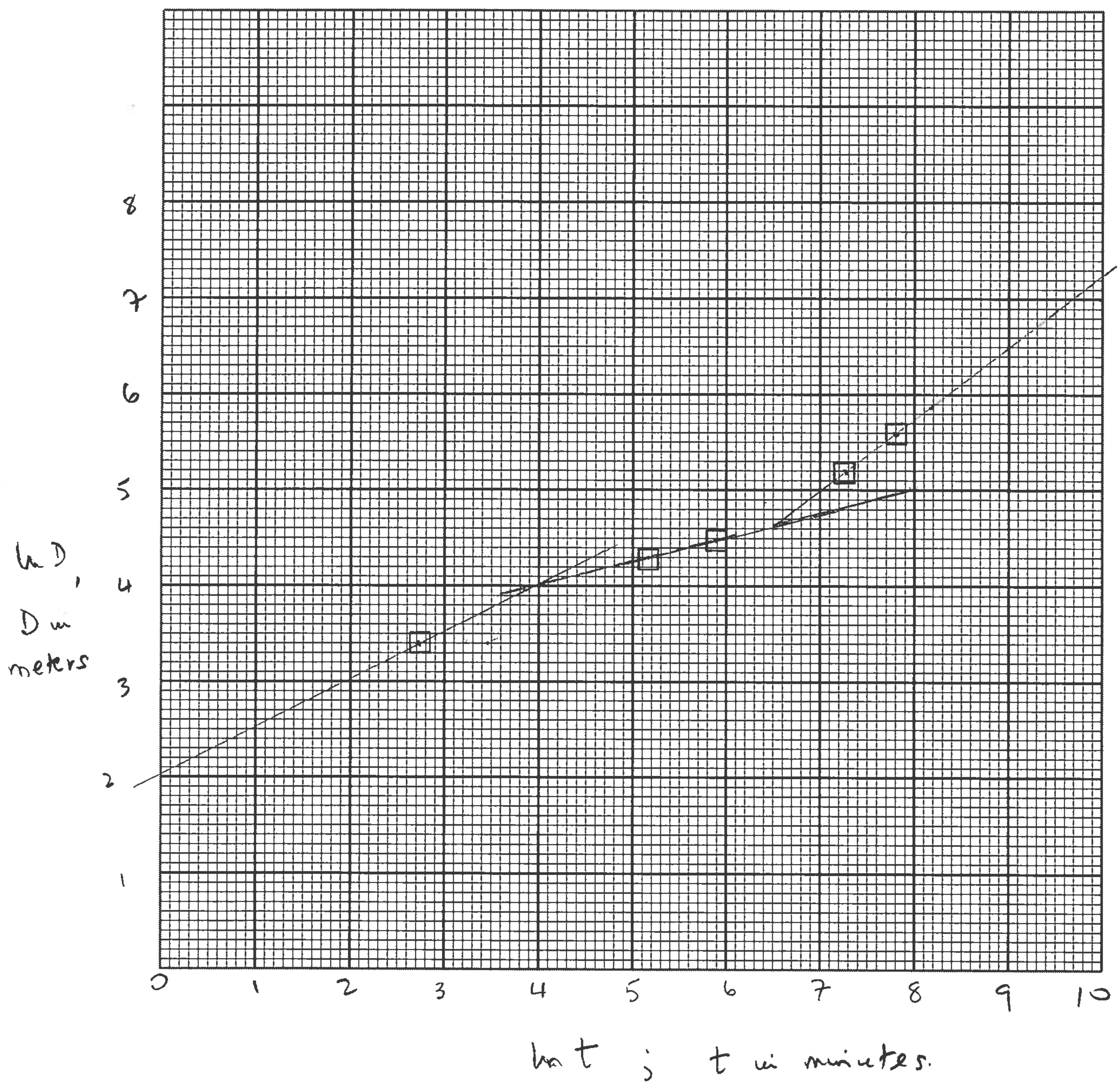
$\ln D = 2.02 \quad \therefore D \sim 7.538\ m$

Volume of oil, $\forall = \frac{\pi D^2}{4} \cdot \delta = \frac{\pi}{4}(7.538)^2(1.4)(10^{-2})$

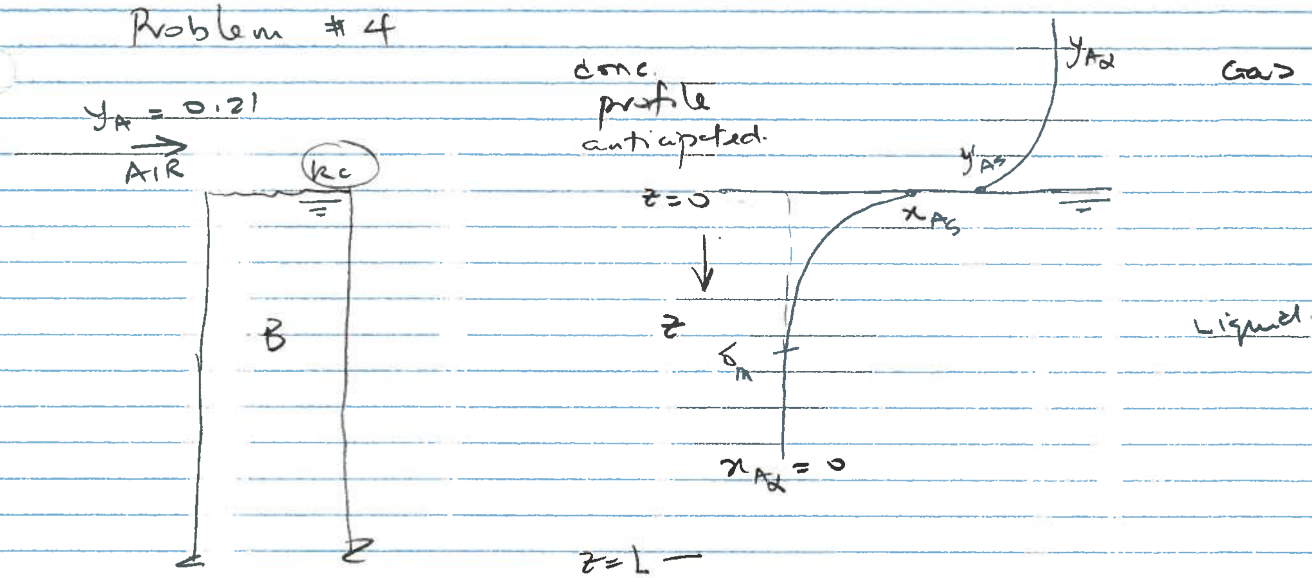
$\doteq 0.625\ m^3$

→

Problem #3



Problem #4



(a) Integral Method: Consider the region $0 \le z < L$

Input + Gen = $\cancel{\text{Output}}^{\,0}$ + Accum. (per unit area)

$$N_A\Big|_{z=0} - \int_0^L k_1 c x_A \, dz = \frac{d}{dt}\left[\int_0^L c x_A \, dz\right]$$

where $N_A = -c D_{AB} \dfrac{dx_A}{dz} + x_A(\cancel{N_A + N_B})$ → 0 (neglect)

$$\therefore \quad N_A\Big|_{z=0} = \int_0^L k_1 c x_A \, dz + \frac{d}{dt}\left[\int_0^L c x_A \, dz\right] \quad (1)$$

This is the integral material balance equation for oxygen (A)

Boundary conditions.

$$z = 0 \qquad -c D_{AB} \frac{dx_A}{dz} = k_c (y_{A\alpha} - y_{As}) \quad (2)$$

$$= k_c K_A (x^*_{A\alpha} - x_{As})$$

$$z = \delta_m \qquad x_A = 0 \quad (3)$$

$$z = \delta_m \qquad \frac{dx_A}{dz} = 0 \quad (4)$$

Assume a concentration profile

$$\frac{x_A}{x_{AS}} = \left(1 - \frac{z}{\delta_m}\right)^2 \quad ; \text{ where } x_{AS}(t) \qquad (5)$$

$$\left.\frac{dx_A}{dz}\right|_0 = x_{AS}\left(-\frac{2}{\delta_m} + \cancel{\frac{2z}{\delta_m^2}}\right) = -\frac{2x_{AS}}{\delta_m} \qquad (6)$$

From b.c.

$$\left.\frac{dx_A}{dz}\right|_{z=0} = -\frac{k_c K_A}{c\mathcal{D}_{AB}}\left(x_{Ad}^* - x_{AS}\right) \qquad (7)$$

$$\frac{2x_{AS}}{\delta_m} = \frac{k_c K_A}{c\mathcal{D}_{AB}}\left(x_{Ad}^* - x_{AS}\right)$$

or

$$\frac{x_{AS}}{x_{Ad}^* - x_{AS}} = \beta\delta_m \quad ; \quad \beta = \frac{k_c K_A}{2c\mathcal{D}_{AB}} \qquad (8)$$

Into equation (1), substitute for x_A from eq. (5)

$$k_c K_A\left(x_{Ad}^* - x_{AS}\right) = k_1 c x_{AS}\int_0^{\delta_m}\left(1 - \frac{z}{\delta_m}\right)^2 dz + \frac{d}{dt}\left[c x_{AS}\int_0^{\delta_m}\left(1 - \frac{z}{\delta_m}\right)^2 dz\right]$$

where the upper limits of the integral has been changed from L to δ_m, the penetration depth.

$$k_c K_A\left(x_{Ad}^* - x_{AS}\right) = k_1 c x_{AS}\delta_m\int_0^1 (1-\eta)^2 d\eta + \frac{d}{dt}\left[c x_{AS}\delta_m\int_0^1 (1-\eta)^2 d\eta\right] \qquad (9)$$

$$k_c K_A \left(x^*_{A\alpha} - x_{AS}\right) = k_1 c\, x_{AS} \frac{\delta_m}{3} + \frac{d}{dt}\left[c\, x_{AS} \frac{\delta_m}{3}\right] \qquad (10)$$

$$\frac{3 k_c K_A}{c} \frac{\left(x^*_{A\alpha} - x_{AS}\right)}{x^*_{A\alpha}} = k_1 \frac{x_{AS}\,\delta_m}{x^*_{A\alpha}} + \frac{d}{dt}\left[\frac{x_{AS}\,\delta_m}{x^*_{A\alpha}}\right] \qquad (11)$$

using equation (8)

$$1 - \frac{x_{AS}}{x^*_{A\alpha} - x_{AS}} = \frac{x^*_{A\alpha}}{x^*_{A\alpha} - x_{AS}} = 1 - \beta\delta_m$$

$$\frac{x_{AS}}{x^*_{A\alpha}} = -\frac{\beta\delta}{1-\beta\delta}$$

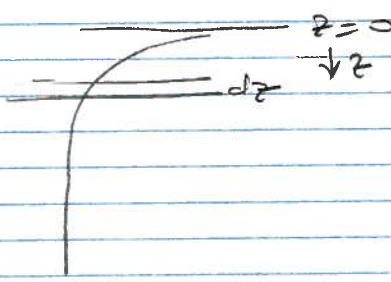
$$\frac{3 k_c K_A}{c}\left(\frac{1}{1-\beta\delta_m}\right) + k_1\left(\frac{\beta\delta_m^2}{1-\beta\delta_m}\right) + \frac{d}{dt}\left[\frac{\beta\delta_m^2}{1-\beta\delta_m}\right] = 0 \qquad (12)$$

b.c. $t = 0, \quad \delta_m = 0$

solve for δ_m and substitute into eq. (5) for concentration profile.

(b) Differential method.

Consider a differential element dz



Balance on A, per unit area, is

$$N_A\Big|_z - k_1 c x_A dz = N_A\Big|_{z+dz} + \frac{d}{dt}(c x_A dz)$$

$$-k_1 c x_A dz = \frac{\partial N_{Az}}{\partial z} dz + \frac{d x_A}{dt}(c\,dz)$$

$$-k_1 x_A = -\frac{\partial}{\partial z}\left(D_{AB}\frac{d x_A}{dz}\right) + \frac{d x_A}{dt}$$

or $$D_{AB}\frac{\partial^2 x_A}{\partial z^2} - k_1 x_A = \frac{\partial x_A}{\partial t}$$

This is the mass transfer equation, subject to the conditions:

$$z = 0 \qquad -cD_{AB}\frac{dx_A}{dz} = k_c K_A (x^*_{A\alpha} - x_A)$$

$$z = \alpha \quad , \quad x_A = 0$$

$$t = 0 \quad , \quad x_A = 0$$

$\longrightarrow$