University of Calgary
Department of Chemical & Petroleum Engineering

aJ

ENCH 501: Transport Processes

Final Examination, Fall 2005

Time: Noon - 3 pm **Wednesday, December 14, 2005**

Instructions: **Attempt All Questions.**
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Problem #1 (30 points)

Pasteurization is the process of heating food (milk and other dairy products, wine, juices, beer, honey, eggs and canned items) for the purpose of killing or inactivating harmful microorganisms or particles. These include bacteria, viruses, protozoa, molds and yeast. Heating makes the product safe for human consumption and increases shelf-life. Unlike sterilization which aims to destroy all the organisms, pasteurization of milk tries to achieve a 5 log reduction (10^{-5} times the original count) in number of viable organisms, particularly common pathogens such as *Mycobacterium tuberculosis* and *Coxiella burnetii*. Milk pasteurization, however, damages or destroys enzymes such as lipases (to digest fats), lactase (to digest milk sugar), phosphatase (for calcium absorption), heat sensitive vitamins C and B_6, and bacteria useful for digestion and boosting immunity.

Pasteurization of milk can be carried out in a number of ways but the standard is the continuous-flow, high-temperature-short-time (HTST) method. The milk is heated from about 4°C to ≥72°C and held at such a temperature for 16s before being cooled back down to below 7°C.

The flow diagram for a pilot system is shown below in Fig. 1. Water is used for both heating and cooling. The heat exchangers (regenerator, heater and cooler) are all single tube-in-shell types and the holding tube is 1inch nominal diameter schedule 40 stainless steel. It is in the holding tube that all milk particles must be kept for 16s at a temperature of at least 72°C and the maximum temperature difference between two points at a cross-section is not to exceed 0.5°C. The residence times (based on average flow velocity) in each unit versus the bulk mean temperatures are shown in Fig. 2. Whole milk from the balance tank enters the system at a volumetric rate of 1.36 US gallons per minute. Heating water is available at 96°C and cooling water at 3°C.

a) Estimate the heat exchange rates in the heater and in the cooler. How long should the holding tube be?

b) If it is assumed that both the velocity and temperature profiles are fully developed in the holding tube, what is the heat flux from its exposed surface into the air in the room?

c) With the same assumptions as for part b), what are the temperatures at the centre and at the inside wall of the holding tube at its inlet and exit? Comments on your results.

Data:

Properties of water (av.) - $\rho = 995.6$ kg.m^3 ; $C_p = 4.186$ kJ/kg K ;
$\mu = 0.8$ mPa s ; k = 0.615 W/m K

Properties of whole (homogenized) milk - $\rho = 1032$ kg/m^3 ; $C_p = 3.9$ kJ/kg K ;
$\mu = 2.1$ mPa s ; k = 0.58 W/mK

Conversions: 1 inch = 25.4 mm ; 1 US gallon = 3.78 litres; 1" sch 40 steel - o.d. 1.315", i.d. 1.049"

FIG 1

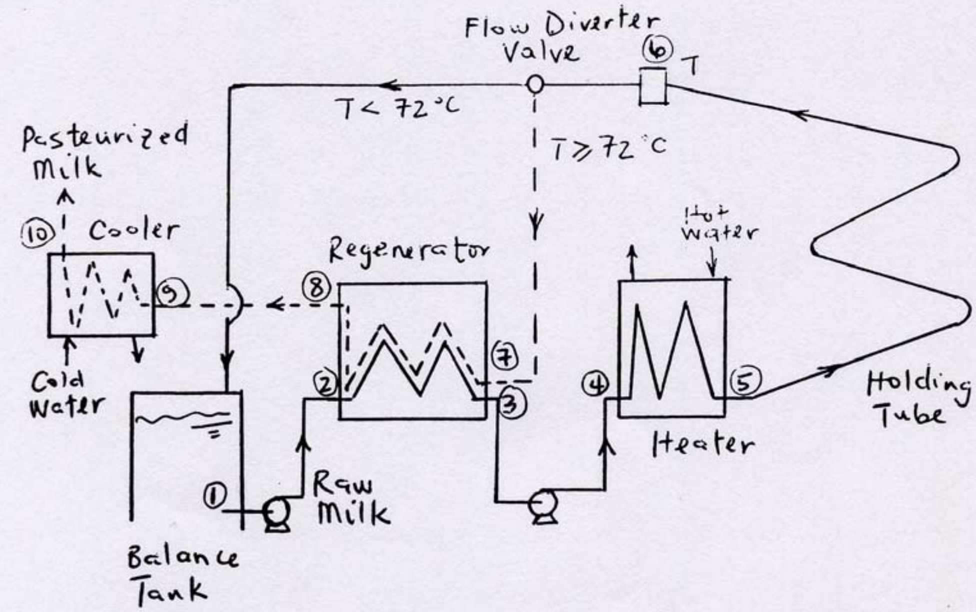
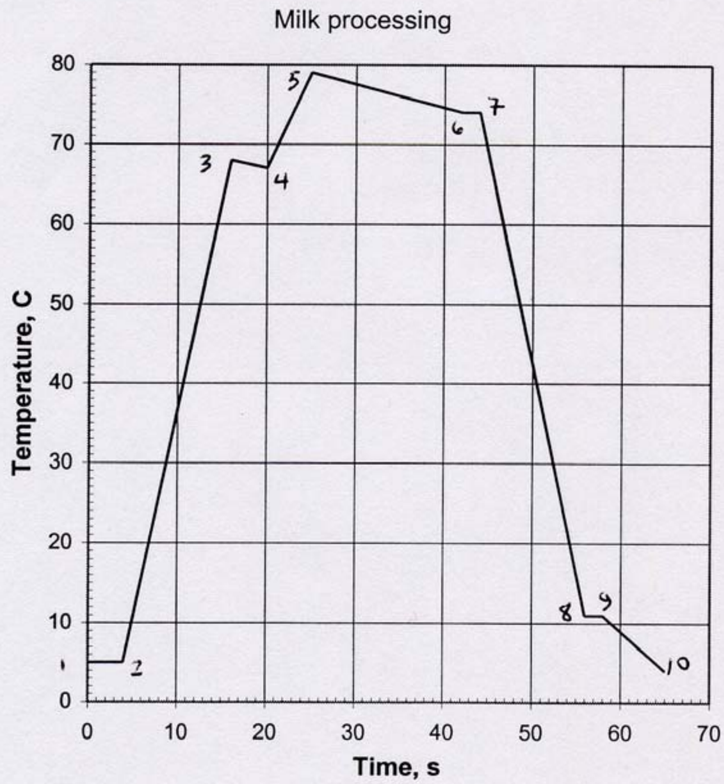


FIG 2

Location	Time, s	Temp, C
1	0	5
2	4	5
3	16	68
4	20	67
5	25	79
6	42	74
7	44	74
8	56	11
9	58	11
10	65	4



Problem #2 (20 points)

In some types of viscometers, a fluid is contained inside the annular space between two concentric vertical cylinders. One of the cylinders is rotated while the second is kept stationary. A torque (force at wall x radius) has to be maintained at the stationary wall to keep it from rotating. From the measurement of the torque, the viscosity of the fluid within the gap can be determined.

Two concentric, vertical cylinders have a liquid (density ρ and viscosity μ) between the gap. The radius of the outside surface of the inner cylinder is $\mathbf{R_1}$ and the radius of the inner surface of the outside cylinder is $\mathbf{R_2}$. If the inner cylinder is rotated at a constant angular velocity ω,

a) derive a relationship for the velocity profile within the gap. State all assumptions and show your steps.

b) Derive an expression for the torque on the outside cylinder.

Problem #3 (30 points)

Carbon dioxide (CO_2) is one of the "green house" gases implicated in global warming. The gas is produced in large quantities from burning fossil fuel to generate electricity in power plants (39% of total), from refineries of crude oil and other industrial processes, from vehicular emissions, cooking and home heating, and animal respiratory and metabolic activities. There is now a strong interest in the capture, storage and sequestration of the compound. Much of the CO_2 produced can be stored in biomass, geological formations and the oceans. One of the many types of geological formations suitable for sequestration contains brine in the pore spaces of sand, i.e. saline formation. A pilot study is being conducted by contacting a gas rich in CO_2 with the flat surface of a soil layer saturated with brine. The brine just covered all the sand grains at the surface. The diffusivity of CO_2 in brine is $1.5(10^{-7})$ m^2/s.

Flue gas from a power plant contains 12 mole % CO_2. This is concentrated in a process to 90%, the balance being nitrogen gas. This gas was raised to a pressure of 5 atm and contacted with a deep layer of soil fully saturated with brine. An impermeable wall is at the bottom of the layer. The porosity of the soil is 0.32. The concentration of the CO_2 in the top layer of the "saline formation" is maintained constant at 135 moles/m^3 solution. As the CO_2 diffuses through the medium, a portion of it reacts with water by a first order reaction ($r_A = -k_1 C_A$, where C_A is the local concentration of CO_2) with a reaction constant $k_1 = 0.0035 s^{-1}$ to form carbonic acid. You may assume the brine and the sand grains are stationary and the brine concentration is 54.41 moles/litre solution .

a) Use the ***integral method*** to derive an expression for the concentration of unreacted CO_2 in the brine within the pore spaces as a function of distance to the surface. Show all important steps and state the assumptions you consider valid.

b) After how long will dissolved (unreacted) CO_2 reach a depth of 1cm?

c) How much CO_2 has been absorbed by the medium by this time?

Problem #4 (20 points)

A bronze sphere is hollow. The inner and outer radii are 6 and 8cm respectively. Ninety percent (90%) of the volume inside the sphere is filled with water and both the bronze and water are initially at 20°C. The rest of the volume consist of small bubbles of air, uniformly dispersed. The suspension is continuously agitated. At time t = 0, the sphere is placed in cold air at -40°C in a chamber. The convective heat transfer between the external surface of the sphere and the cold air is given as 21 W/m^2K.

It is assumed that the suspension in the sphere is well mixed until ice formed at 0°C, and that the ice particles, when formed, are uniformly dispersed until all the liquid water disappeared.

Estimate how long it takes to cool the system to -30°C. Neglect temperature gradients in the system.

Data: Thermophysical properties

Material	Density, kg/m^3	Specific Heat, kJ/kgK	Thermal Conductivity, W/mK
Bronze	8,666	0.343	26
Water	1,000	4.217	0.575
Ice	920	2.050	2.2

Heat of fusion (water to ice) = 333.4 kJ/kg

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#1 Flow rate of whole milk is 1.36 US gals / min

i.e. $Q = \frac{1.36(3.78)}{60(1000)} = 8.568(10^{-5})\ m^3/s$

The holding tube i.d. = 1.049 inches or 0.026645 m

$\therefore$ Av. vel. in holding tube $= \frac{Q}{A} = \frac{4Q}{\pi D^2}$

$$= \frac{4(8.568)(10^{-5})}{\pi(0.026645)^2}$$

$$\bar{u} = 0.15366\ m/s.$$

The Reynolds number in the holding tube =

$$\frac{D\bar{u}\rho}{\mu} = \frac{(0.026645)(0.15366)(1032)}{2.1(10^{-3})}$$

$= 2012.1$, i.e. flow is laminar.

(a) The heater is pt. 4 → 5, i.e. $\Delta T = 79 - 67$

Heat exchange, $E_H = \dot{m} C_p \Delta T_{bm} = \rho Q C_p \Delta T$

$$E_H = (1032)(8.568)(10^{-5})(3900)(12)$$

$$= 4,138.14\ W \longrightarrow$$

The cooler is pt. 9 → 10, i.e. $\Delta T = (4 - 11)$

$$E_c = (1032)(8.568)(10^{-5})(3900)(7)$$

$$= 2,413.91\ W \longrightarrow$$

The maximum velocity in the holding tube is $2\bar{u}$

For a minimum 16 s residence time, the length of the tube $L \geq (2\bar{u})(16)$

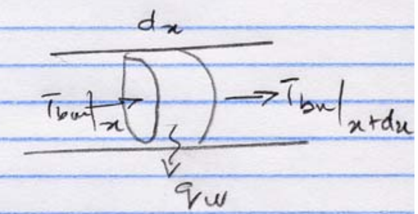
$$= 2(0.15366)(16) = 4.917\,m$$

⟶

(b) The holding tube is section $5 \rightarrow 6$ and the bulk mean temperature profile appears linear

$$\frac{dT_{bm}}{dx} = \frac{74-79}{4.917} = -1.0169 \ {}^{\circ}C/m$$

From an energy balance on a disc



$$\frac{dT_{bm}}{dx} = \frac{2R_o q_w}{R_i^2 \bar{u} \rho C_p}$$

$$2\pi R_o dx\, q_w = \dot{m}\, C_p \frac{dT_{bm}}{dx}$$

where R_i and R_o are internal and external radii of the pipe.

Given

$$R_o = \frac{1.315}{2}\frac{(2.54)}{100} = 0.0167\,m; \quad R_i = \frac{1.049}{2}\frac{(2.54)}{100} = 0.0133\,m$$

$$\therefore \quad q_w = 3341.9 \quad W/m^2$$

⟶

(c) from Notes (p. 169)

$$T_{bm} = T_o + \frac{7}{96}\frac{U_{max} R^2}{\alpha}\frac{dT_{bm}}{dx}; \quad \alpha = \frac{k}{\rho C_p}$$

where T_o is the temp along the axis.

~~At~~ exit, substitute value at pt. 6

$$74 = T_o + \frac{7}{96}\frac{(0.15366)2}{\alpha}\frac{(0.026645)^2}{4}(-1.0169)$$

$$\alpha = \frac{0.58}{(1032)(3900)} = 1.441(10^{-7})\ m^2/s$$

Substitute

$$T_o = 74 + 28.07 = 102.07\ °C$$

Also since

$$T_w = T_o + \frac{3}{16}\frac{u_{max}R^2}{\alpha}\frac{dT_{bm}}{dx}$$

$$= 102.07 - 72.17 = 29.9\ °C$$

At inlet

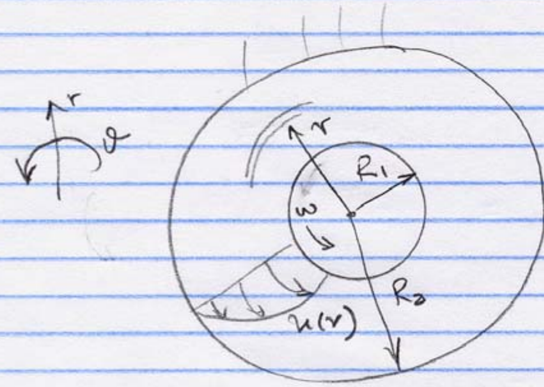
$$T_o = 79 + 28.07 = 107.07\ °C$$

$$T_w = 107.07 - 72.17 = 34.9\ °C$$

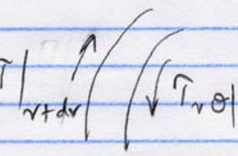
Comment:

These are large temperature differences. The assumptions of fully developed temperature and velocity profiles cannot be valid if in the actual system, the maximum temperature difference at any cross-section is only 0.5 °C.

#2



Perform a force balance on a differential element - a ring of thickness dr at r. Positive θ-direction is anti-clockwise.



Height of ring = dz. Only force is shear.

Since rate of change of momentum = 0

$$\left.\tau(2\pi r\,dz)\right|_r - \left(\tau(2\pi r\,dz)\right)_{r+dr} = 0$$

Hence

$$\frac{d(\tau r)}{dr} = 0 \quad ; \quad \tau = -\mu\frac{du}{dr}$$

$$\therefore \quad \frac{d\left(r\frac{\partial u}{\partial r}\right)}{dr} = 0$$

Integrate once

$$r\frac{du}{dr} = C_1 \quad \text{or} \quad \frac{du}{dr} = \frac{C_1}{r}$$

Integrate second time

$$u = C_1 \ln r + C_2$$

Apply b.c. $r = R_1$, $u = R_1\omega$

$r = R_2$ $u = 0$

$$\left.\begin{aligned} R_1\omega &= C_1 \ln R_1 + C_2 \\ 0 &= C_1 \ln R_2 + C_2 \end{aligned}\right\}$$

$$R_1 \omega = c_1 \ln\left(R_1/R_2\right) \Rightarrow c_1 = \frac{R_1 \omega}{\ln\left(R_1/R_2\right)}$$

$$c_2 = -c_1 \ln R_2 \Rightarrow c_2 = -\frac{R_1 \omega}{\ln\left(R_1/R_2\right)} \ln R_2$$

$$\therefore \quad u = c_1 \ln r - c_1 \ln R_2 = c_1 \ln\left(r/R_2\right)$$

$$\therefore \quad u = \frac{R_1 \omega}{\ln\left(R_1/R_2\right)} \ln\left(r/R_2\right)$$

$\longrightarrow$

$$\frac{du}{dr} = \frac{c_1}{r} = \frac{\omega}{\ln\left(R_1/R_2\right)}\left(\frac{R_1}{r}\right)$$

$\therefore$ shear stress at stationary wall

$$\tau\Big|_{R_2} = -\mu \frac{du}{dr}\Big|_{R_2} = -\mu\omega \frac{\left(R_1/R_2\right)}{\ln\left(R_1/R_2\right)}$$

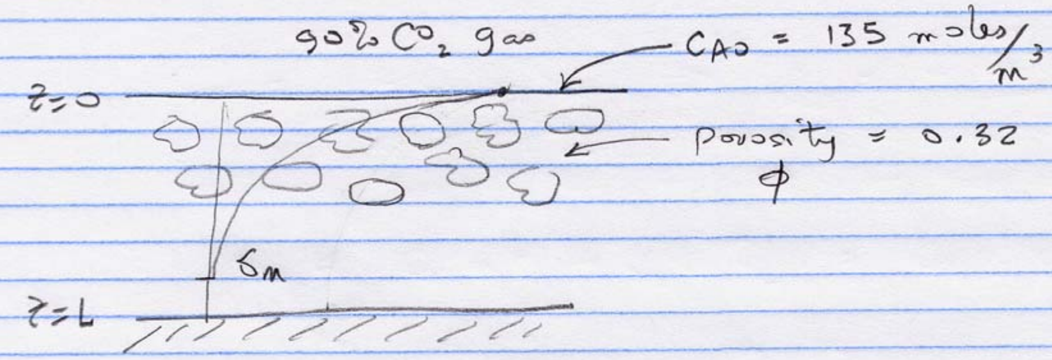
The torque, $\Gamma = \tau\big|_{R_2} \cdot$ Area

$$\Gamma = -2\pi R_2 L\left(\mu\omega\right) \frac{R_1/R_2}{\ln\left(R_1/R_2\right)} > 0$$

since $\ln\left(R_1/R_2\right) < 0$

$\longrightarrow$

Problem # 3



This is a problem of diffusion with chemical reaction. The domain is semi-infinite.

Perform a material balance on CO_2 in $0 \le z \le L$

Input + Generation = Output + Accum

$$\text{Output} \to 0$$

$$N_A\Big|_{z=0} - \int_0^L k_1 C_A \phi \, dz = \frac{d}{dt}\left[\int_0^L C_A \phi \, dz\right] \quad ; \; \phi = \text{porosity} \qquad (1)$$

The flux is defined as

$$N_A = -c\mathcal{D}_{AB}\frac{dx_A}{dz} + x_A\left(N_A + N_B + N_C\right)$$

where $A = CO_2$, B = brine and C = carbonic acid.

The brine conc. = 54.41 (1000) mols/m^3

$$\therefore \quad x_A \sim \frac{135}{54.41(1000)} \simeq 0.00248 \ll 1$$

Note that $x_A \ll 1$ is more important than $N_B = N_C = 0$

$\therefore$ neglect convection and assume $c \simeq$ const

$$\therefore \quad N_A \doteq -\mathcal{D}_{AB}\frac{dC_A}{dz} \qquad (2)$$

Subst. (2) into (1)

$$-\mathcal{D}_{AB}\frac{dC_A}{dz}\Big| - \int_0^L k_1 C_A \phi \, dz = \frac{d}{dt}\left[\int_0^L C_A \phi \, dz\right] \qquad (3)$$

Conditions for the problem

$$z = 0 \qquad C_A = C_{A0} \qquad \text{const. b.c.}$$

$$z = \delta_m \qquad \frac{dC_A}{dz} = 0 \qquad (\text{no flux})$$

$$z = \delta_m \qquad C_A = 0$$

Assume a profile $\quad C_A = a + bz + cz^2$

$$\frac{dC_A}{dz} = b + 2cz$$

use conditions

$$\left.\begin{aligned} 0 &= b + 2c\delta_m \\ 0 &= a + b\delta_m + c\delta_m^2 \\ C_{A0} &= a \end{aligned}\right\} \qquad \begin{aligned} a &= C_{A0} \\ c &= a/\delta_m^2 \\ b &= -2c\delta_m \end{aligned}$$

$$\therefore \quad C_A = C_{A0} - 2C_{A0}\frac{z}{\delta_m} + \frac{C_{A0} z^2}{\delta_m^2} = C_{A0}\left(1 - 2\frac{z}{\delta_m} + \frac{z^2}{\delta_m^2}\right)$$

$$\frac{C_A}{C_{A0}} = \left(1 - \frac{z}{\delta_m}\right)^2 \qquad (4)$$

substitute eq. (4) into integral eq. (3)

$$\int_0^L k_1 C_A \phi\, dz \doteq k_1 \phi C_{A0} \int_0^{\delta_m} \frac{C_A}{C_{A0}}\, dz = k_1 \phi C_{A0} \delta_m \int_0^1 \frac{C_A}{C_{A0}} \frac{dz}{\delta_m}$$

$$= k_1 \phi C_{A0} \delta_m \int_0^1 (1 - 2\eta + \eta^2)\, d\eta \quad ; \quad \eta = z/\delta_m$$

$$= k_1 \phi C_{A0} \delta_m \left[\eta - \eta^2 + \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{3} k_1 \phi C_{A0} \delta_m$$

$$\int_0^L C_A \phi\, dz \simeq \phi \int_0^{\delta_m} C_A\, dz = \phi C_{A0} \delta_m \int_0^1 \frac{C_A}{C_{A0}} \frac{dz}{\delta_m} = \frac{1}{3}\phi C_{A0} \delta_m$$

$$\left.\frac{dC_A}{dz}\right|_{z=0} = b = -2\frac{C_{Ao}}{\delta_m^{\cancel{2}}}\cancel{\delta/_m} = -\frac{2C_{Ao}}{\delta_m}$$

Subst. into (3)

$$\frac{2C_{Ao}}{\delta_m}D_{AB} - \frac{1}{3}k_1\phi C_{Ao}\delta_m = \frac{d}{dt}\left[\frac{\phi}{3}C_{Ao}\delta_m\right]$$

$$6D_{AB} - k_1\phi\delta_m^2 = \phi\delta_m\frac{d\delta_m}{dt} = \frac{\phi}{2}\frac{d\delta_m^2}{dt}$$

The o.d.e.

$$\frac{d\delta_m^2}{dt} = \frac{12D_{AB}}{\phi} - 2k_1\delta_m^2 \qquad (5)$$

is solved subject to i.c. $t=0$, $\delta_m=0$

or $\ln\left[\frac{12D_{AB}}{\phi} - 2k_1\delta_m^2\right] = -2k_1t + C$

and $C = \ln\left[\frac{12D_{AB}}{\phi}\right]$

$$\ln\left[1 - \frac{2k_1\delta_m^2\phi}{12D_{AB}}\right] = -2k_1t$$

$$\frac{k_1\phi\delta_m^2}{6D_{AB}} = 1 - e^{-2k_1t}$$

$$\delta_m = \sqrt{\frac{6D_{AB}}{k_1\phi}}\left(1 - e^{-2k_1t}\right)^{\frac{1}{2}} \qquad (6)$$

$\longrightarrow$

The profile is eq. (4) with eq. (6).

(b) Substitute values into eq. (6)

$$\frac{(0.0035)(0.32)(0.01)^2}{6(1.5)(10^{-7})} = 1 - e^{-2(0.0035)t}$$

$$t = 18.985 \text{ s}$$

$\longrightarrow$

(c) Total uptake of CO_2

$$\Upsilon = \int_0^{\tau} N_A\big|_{z=0}\, dt \quad ; \quad \tau = 18.985\text{s}$$

$$= +D_{AB} \int_0^{\tau} \frac{2C_{A0}}{\delta_m}\, dt$$

$$= 2\, C_{A0} D_{AB} \int_0^{\tau} \frac{1}{\delta_m}\, dt$$

$$\beta = \sqrt{\frac{6 D_{AB}}{k_1 \phi}}$$

$$= \frac{2 C_{A0} D_{AB}}{\beta} \int_0^{\tau} \frac{1}{(1 - e^{\gamma t})^{\frac{1}{2}}}\, dt \quad ; \quad \gamma = -2k_1$$

Let $\psi = (1 - e^{\gamma t})^{\frac{1}{2}}$, $d\psi = \frac{1}{2}(1 - e^{\gamma t})^{-\frac{1}{2}}(-\gamma e^{\gamma t})\,dt$

$$= \frac{1}{2\psi}(-\gamma)(1 - \psi^2)$$

$$\int_0^{\tau} \frac{1}{(1 - e^{\gamma t})^{\frac{1}{2}}}\, dt = \int_0^{\psi} \left(-\frac{1}{\gamma}\right)\frac{1}{\cancel{\psi}} \frac{2\cancel{\psi}}{1 - \psi^2}\, d\psi = \int_0^{\psi} \left(-\frac{2}{\gamma}\right) \frac{d\psi}{1 - \psi^2}$$

$$\Upsilon = \frac{2 C_{A0} D_{AB}}{\beta}\left(-\frac{2}{\gamma}\right)\left[\frac{1}{2}\ln\left(\frac{1+\psi}{1-\psi}\right)\right]_0^1$$

$$= \frac{\cancel{2} C_{A0} D_{AB}}{\beta k_1}\cdot\frac{1}{\cancel{2}}\ln\left(\frac{1+\psi}{1-\psi}\right)$$

$$= \frac{C_{A0} D_{AB}}{k_1}\frac{(k_1\phi)^{\frac{1}{2}}}{(6 D_{AB})^{\frac{1}{2}}}\ln\left(\frac{1+\psi}{1-\psi}\right)$$

$$= C_{A0}\sqrt{\frac{D_{AB}\,\phi}{6k_1}}\ \ln\left(\frac{1+\psi}{1-\psi}\right)$$

For $t = 18.985$ s

$$\psi = \left(1 - e^{-2k_1 t}\right)^{\frac{1}{2}} = \left(1 - e^{-2(0.0035)(18.985)}\right)^{\frac{1}{2}}$$

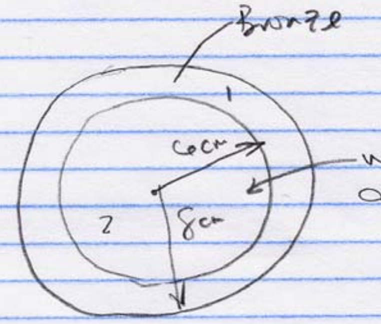
$$= 0.352765$$

$$\Upsilon = 135\sqrt{\frac{1.5(10^{-7})(0.32)}{6(0.0035)}}\ \ln\left(\frac{1.352765}{0.647235}\right) \quad \frac{\text{moles}}{m^2 \text{ surface area}}$$

Total CO_2 uptake $= 0.1505$ moles$/m^2$ surface area

$\longrightarrow$

4



Use lumped heat capacity method.

- Ignore heat gain or release by bubbles
- $V_{water} = 0.9 V_{cavity}$

Problem has 3 stages – cool water from 20°C to 0°C.
– freeze the water at 0°C
– cool mass to −30°C

from Notes: The general solution is

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\frac{hA_1}{\rho_1 \forall_1 C_{p_1} + \rho_2 (0.9)\forall_2 C_{p_2}}\right] \quad (1)$$

for the composite body.

$A_1 = 4\pi R_1^2 = 4\pi(0.08)^2 = 0.080425\ m^2$

$h_\infty = 21\ W/m^2 K$

$\rho_1 \forall_1 C_{p_1} = 8666\left(\frac{4}{3}\pi\left(0.08^3 - 0.06^3\right)\right)343$

$= 3,685.47\ J/K$

For stage 1, (2) is liquid water

$(0.9)\forall_2 \rho_2 C_{p_2} = 0.9(1000)\left(\frac{4}{3}\pi(0.06)^3\right)(4217)$

$= 3,433.9\ J/K$

Substitute values into (1)

$$\frac{0+40}{20+40} = \exp\left[-\frac{21(0.080425)t}{3685.47+3433.9}\right]$$

$$t_1 = 1709.17\ s$$

Stage 2 As the water freezes, the ratio of volume of water to air changes but total mass remains constant. In stage 2, the latent heat of fusion is removed.

$$\text{Total} = \underline{0.9\,\rho_2 \forall_2}\,\Delta H_f$$

$$= (0.9)(1000)\left(\frac{4}{3}\pi(0.06)^3\right)(333.4)\ kJ$$

$$= 271.49\ kJ$$

But rate of heat removal $= h A_o \Delta T$

$$= 21(0.080425)(40) = 67.557\ W$$

$$\therefore \text{time to freeze} = \frac{271.49\,(1000)}{67.557}$$

$$t_2 = 4{,}018.65\ s$$

Stage 3 The water is now all frozen.

Mass of water = mass of ice $= 0.9\,\forall_2 \rho_2$

$$\frac{T-T_2}{T_0-T_2} = \exp\left[-\frac{21(0.080425)t}{\rho_1 \forall_1 c_{p_1} + \left[\rho_2(0.9)\forall_2\right]c_{p_{ice}}}\right]$$

$$\frac{-30-(-40)}{0-(-40)} = \exp\left[-\frac{21(0.080425)t}{3685.47+1669.3}\right]$$

$$\frac{10}{40} = \exp\left[-0.000315\,t\right]$$

$$t_3 = 4{,}395.28$$

$\therefore$ Total time $= t_1 + t_2 + t_3$

$$= (1709.17 + 4{,}018.65 + 4{,}395.28)\,s$$

$$= 10{,}123.1\ s \quad \text{or} \quad 2.812\ \text{hours}$$