G. J. Dec. 3, 2001

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Mathematical Methods in Chemical Engineering

Final Examination, Fall 2001

Time: 12 noon - 3 pm **Thursday, December 13, 2001**

Instructions: **Attempt All Questions.**
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Problem #1 (25 points)

In the analytical laboratory of a manufacturing company, a novice employee had inadvertently left a 60% full beaker of pure benzene on a work bench at the end of a work day. The beaker is a flat-bottomed cylinder, 160 mm tall and 100 mm inside diameter. The beaker was covered by a wire gauze (or net) which kept dust out but did not hinder the exit of benzene vapour. The ventilation system of the room is routinely turned off at 5pm (the end of a work day) and turned on again at 8am next morning. A small fan which moves air towards the direction of the beaker and was left on. Air was thus blown over the mouth of the beaker continuously. There were no convection currents in the space within the beaker above the liquid. The liquid was also saturated with air.

The room is 3m x 4m x 2.5m and 20% of the volume is occupied by equipment, furniture and bottles of chemicals. The room temperature and pressure are constant at 16.85°C (or 290K) and 680 mm Hg respectively. The liquid benzene was always maintained at the room temperature. Vaporized benzene is mixed with the air in the room and neither could escape.

An experienced employee enters the laboratory at 6 am the following day and immediately smelled the benzene. By this time, the liquid level had dropped by 2.1 mm. Given only the data below;

(a) Estimate the diffusion coefficient for benzene vapour in air;

(b) If the acute toxicity limit for benzene in air is 2000 ppm (molar basis), should the employee immediately leave the room because the benzene in the air is at or above this level? Show your calculations.

(c) How long after the benzene was left on the bench would the benzene in air attain the acute toxicity limit? How much liquid (in m^3) would have been evaporated? Assume the ventilation stayed off throughout the period and no doors or windows were open.

Data: Properties of Benzene

Mol. Wt. (kg/kmol)	78.11	***Liquid density*** (kg/m^3)	891.3
Vapour pressure @ 290K (kPa)	8.77		

Universal Gas Constant, R (kJ/kmol K)	8.314	(Litre atm/mol K)	0.08205

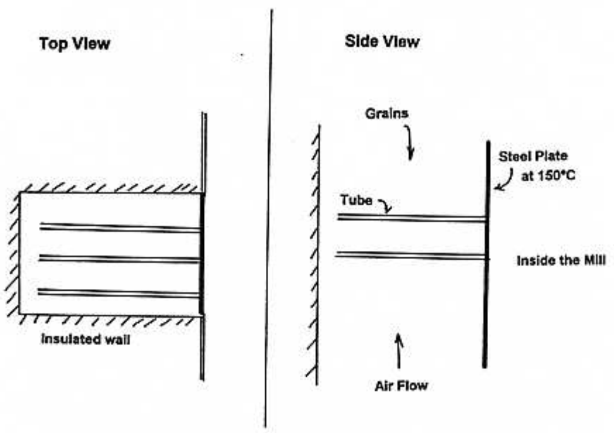
Problem #2 (25 points)

In Europe where land is a premium, the trend is towards erecting multiple-use buildings for commercial operations. A seven-story building, for example, may have the top 2 floors used for raising chickens and pigs; the next 2 floors down for abatoirs. In the next floor down or the 3rd floor, the combined animal waste and unuseable parts are digested with micro-organisms to produce fertilizers. The latter is moved down to the 2nd floor and used to grow vegetables and cultivate mushrooms. The ground floor is open to the public as a retail outlet for eggs, meat, mushrooms and fresh vegetables.

Learning about such operations gave Liz Smart, an entrepreneur, an idea that grains (such as wheat and corn) can be dried by recovering and using heat from a local steel mill. This heat will normally have been lost to the ambient. The idea is as follows. Knock out a part of the wall of the mill near the furnace. Install a vertical steel plate, 4m wide by 10m tall, in the wall which is then maintained at a constant temperature of 150°C by the hot gases inside the mill. Attach 7500 copper tubes, each 5cm outside diameter, 5mm wall thickness and 1.5m long to the face of the steel plate in a staggered fashion up and down the wall. The open end of the tube is plugged with cotton, an insulator. Enclose the tubes within insulating walls on the other 3 sides to form a drying tower. (See sketches.) The tubes are horizontal. Grains are fed from above. Air, drawn from the mill at 40°C, is fed from below. Over the tube surfaces, the convective heat transfer coefficient is 23 W/m^{2}K. The properties of copper are as follows; ρ = 8954 kg/m^3; C_p = 0.383 kJ/kgK; k = 386 W/mK.

(a) Estimate the amount of energy available from the tubes and the wall for drying the grains.

(b) If the grain enters the tower with a water content of 15% by weight and it is to be dried to contain 5% water by weight, at what rate would the grains be processed in the tower given that 2.75 MJ of heat removes one kilogram of moisture?



Problem #3 (25 points)

In a pilot plant, light crude oil flowing through a straight tube (2 cm i.d. and 12 m long) is to be heated from 12°C to 375°C before passing the liquid into an atmospheric distillation unit. The oil is placed under sufficiently high pressure that its volume does not increase either by vapour generation or expansion. At the flow rate, the fully developed velocity distribution is given by:

$$u = U_{max}(1 - r/R)^{1/6}\ ; \quad U_{av} = 0.791\ U_{max}$$

where U_{max} is the centre-line velocity and U_{av} is the average velocity in the tube of radius R. The tube is heated by electrical tape wound over its length and insulated on the outside. The tape provided a constant heat flux at the wall. The mass rate of the crude oil is 0.347 kg/s. Given the data below:

(a) derive an expression for the Nusselt number for the system if both velocity and temperature profiles are assumed fully developed.

(b) Estimate the temperature of the oil at the centre of the outlet of the tube.

Data: Properties of crude oil:

ρ = 850 kg/m^3; μ = 6 mPa.s; k = 0.093 W/mK; C_p = 2.328 kJ/kgK

Problem #4 (25 points)

(c) Two caverns used for underground storage are nearly spherical and are located with 300m separating them. The larger cavern, 80m diameter, is filled with liquid propane at 709.1 kPa and 286.9 K. The smaller cavern is 30m diameter and it contains waste nuclear fuels at a temperature of 124°C. The effective thermal conductivity of the soil is 1.21 W/mK.
Estimate the volume rate at which propane vapour must be vented from the larger cavern if the latent heat of vaporization is 355.9 kJ/kg. Vapour density is 15.3 kg/m^3.

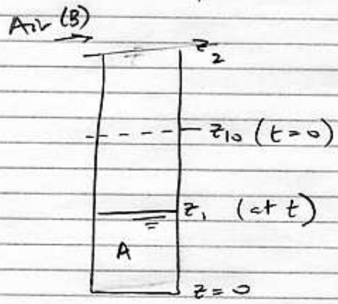
(d) 1.2 litres of water at 7°C was put in a stainless steel sauce pan initially at room temperature of 20°C. The pan and cover have a mass of 750g. The inside diameter and height of the pan are 16cm and 8cm respectively. The sauce pan was then placed on a stove and the coil turned on such that 550 W of energy is continuously supplied to the pot. A gentle breeze also blows over the sides and the cover of the pot such that the heat transfer coefficient is 36 W/m^{2}K. The properties of steel are as follows: ρ = 7833 kg/m^3; C_p = 0.465 kJ/kg K; k = 54 W/mK.
   (i) Estimate the minimum time for the water to boil at 96°C.
   (ii) Why is this the minimum time? Would the mass of water in the container remain constant over the heating period?

Properties of water: ρ = 998 kg/m^3; C_p = 4.12 kJ/kg K; ΔH_v = 2268 kJ/kg

ENCH 501 Final Solution Dec. 13, 2001

Problem #1

From the Notes, p 174 ; Using the pseudo-steady assumption for vaporization of benzene and the diffusion of the vapor through stagnant air,



$$\left(z_1(t) - z_2\right) \frac{dz_1}{dt} = \Gamma = \text{const.} \quad ①$$

where

$$\Gamma = \frac{M_A C D_{AB}}{\rho_L} \ln \frac{y_{B2}}{y_{B1}}$$

solve ①

$$\left(z_1(t) - z_2\right)^2 - \left(z_{10} - z_2\right)^2 = 2\Gamma t \quad ②$$

From data : $z_2 = 160\,mm$; $z_{10} = 0.6 z_2 = 96\,mm$

After 13 hours or 46,800 s, $z = 96 - 2.1$
(from 5pm to 6am) $= 93.9\,mm$

$M_A = 78.11\ kg/kmol$, $C = P/RT$ (assume ideal gas)

where $P = 680\,mm\,Hg$ or $\frac{680}{760} = 0.8947\,atm$

$T = 290\,K$

$$C = \frac{0.8947}{0.08205(290)} = 0.0376\ \frac{mol}{litre} \text{ or } \frac{kmol}{m^3}$$

$\rho_L = 891.3\ kg/m^3$

$y_{B2} = 1 - y_{A2} \simeq 1$

$$y_{B1} = 1 - y_{A1} = 1 - \frac{8.77}{\frac{680}{760}(101.325)} = 0.9033$$

$$\beta = \frac{(78.11)(0.0376)}{891.3} D_{AB} (0.10174) = D_{AB} \times 3.3527 (10^{-4}) \; m^2/s$$

The left side of equation (2) is

$$(0.0939 - 0.16)^2 - (0.096 - 0.16)^2 = 2.7321 (10^{-4})$$

Substitute these values into eq. (2)

$$2.7321 (10^{-4}) = 2 (D_{AB}) (3.3527)(10^{-4})(46,800)$$

(a) $$D_{AB} = 8.706 (10^{-6}) \; m^2/s \longrightarrow$$

(b) Moles of benzene vaporized is

$$\frac{(\Delta z) A \rho_L}{M_A} = \frac{(0.0021) \frac{\pi (0.1)^2}{4} (891.3)}{78.1} = 1.882 (10^{-4}) \; kmol/s$$

$$\frac{\cancel{m} \cdot m^2 \; \frac{\cancel{kg}}{\cancel{m^3}}}{\cancel{kg}} \cdot kmol \qquad \text{or } \underline{0.1882 \; mol/s}.$$

The air plus benzene in the room

$$P V = n R T \quad ; \quad V = 0.8 \left[3 \times 4 \times 2.5 \right] = 24 \; m^3 = 24,000 \text{ litres}$$

Hence
$$n_{air} = \frac{0.8947 (24)(1000)}{0.08205 (290)} = 902.43 \text{ moles}$$

$$\therefore \text{ mole fraction} = \frac{0.1882}{902.43} = 2.0855 (10^{-4}) < 2 (10^{-3})$$

The employee does not have to leave the room immediately.

(c) The amount of benzene in the air at the toxic level is given by

$$\frac{x}{902.43} = 2(10^{-3}) \quad \therefore \quad x = 1.805 \text{ mols}$$

This is equivalent to a liquid volume of

$$\frac{1.805(10^{-3})\ 78.11}{891.3} = 1.5817(10^{-4})\ m^3$$

This is equivalent to a level drop of 0.02014 m or 20.14 mm.

Using equation (2), the time required is given by

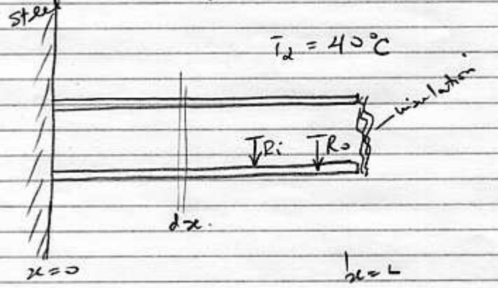
$$(0.07586 - 0.16)^2 - (0.096 - 0.16)^2 = 2(3.3527)(10^{-4})(8.706)(10^{-6})\ t$$

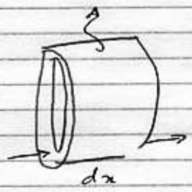
$$t = 511{,}045.76 \text{ s}$$

or 141.96 hours

Problem #2

Consider one tube.





Energy Balance over element

$$\left(-k\frac{dT}{dx}\right)\pi\left(R_o^2-R_i^2\right) = \left(-k\frac{dT}{dx}\right)\pi\left(R_o^2-R_i^2\right) +$$

input

$$\left(-k\pi\right)\left(R_o^2-R_i^2\right)\frac{d^2T}{dx^2}\,dx +$$

$$h\left(2\pi R_o\,dx\right)\left(T-T_\alpha\right)$$

output

or $$\frac{d^2T}{dx^2} = \left(\frac{2R_o}{R_o^2-R_i^2}\,\frac{h}{k}\right)\left(T-T_\alpha\right)$$

or $$\frac{d^2\theta}{dx^2} = \beta\theta \quad ; \quad \theta = T - T_\alpha \qquad (1)$$

$$\beta = \frac{2R_o}{R_o^2-R_i^2}\cdot\frac{h}{k}$$

Subject to

$x=0 \quad T = T_w \quad$ or $\quad \theta = \theta_w = T_w - T_\alpha$

$x=L \quad \frac{dT}{dx} = 0 \quad$ or $\quad \frac{d\theta}{dx} = 0$

Solution to (1)

$$\theta = A\sinh\sqrt{\beta}\,x + B\cosh\sqrt{\beta}\,x \qquad (2)$$

Apply conditions

$$\theta = \theta_w \left\{ \cosh\sqrt{\beta}\,x - \tanh\sqrt{\beta}\,L \sinh\sqrt{\beta}\,x \right\}$$

also $$\left.\frac{d\theta}{dx}\right|_{x=0} = -\theta_w \tanh\sqrt{\beta}\,L$$

Rate of heat input into a tube, at the base

$$Q_{tube} = -k(\pi)(R_o^2 - R_i^2)\left.\frac{dT}{dx}\right|_{x=0}$$

$$= k(\pi)(R_o^2 - R_i^2)\,\theta_w \tanh\sqrt{\beta}\,L$$

Total Rate of heat transfer from tube and walls to air

$$Q_{total} = 7500(\pi)(R_o^2 - R_i^2)\,k\,\theta_w \tanh\sqrt{\beta}L \; + \; h\,A_b\,(T_w - T_\infty)$$

where $$A_b = 40 - 7500(\pi)\frac{(0.05)^2}{4}$$
$$= 40 - 14.726 = 25.274\ m^2$$

and $$\sqrt{\beta} = \left[\frac{2(0.025)}{(0.025)^2 - (0.02)^2}\ \frac{23}{386}\right]^{\frac{1}{2}} = 3.6388\ m^{-1}$$

$\tanh\sqrt{\beta}L = 0.999964$ for $L = 1.5\,m$

$$Q_{total} = \underset{(tubes)}{225{,}090.87} + \underset{(wall)}{63{,}943.22}\quad \frac{J}{s}$$

$$= 289{,}034.09\ W$$

Let x = mass of dry matter in grain
y = mass of water

Feed $\frac{y_1}{x+y_1} = 0.15$; Product $\frac{y_2}{x+y_2} = 0.05$

If $x = 1 \frac{kg}{s}$, mass of water removed = $y_1 - y_2$
$y_1 = 0.1765 \frac{kg}{s}$ and $y_2 = 0.05263$

$\therefore$ $y_1 - y_2 = 0.12384 \frac{kg}{s}$ of water removed.

This is equivalent to 0.340557 MJ of energy used.
Since energy supply is 0.289 MJ/s , the supply rate of dry grain, $x_{feed} = \frac{0.289}{0.34} = 0.8487 \frac{kg}{s}$

This contains $(0.1765)(0.8487)$ kg/s of water in feed grain

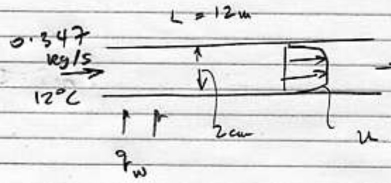
or 0.1498 kg

$\therefore$ Mass of wet grain processed $= 0.9985$ kg/s
(feed)

$\longrightarrow$

Problem #3

A problem similar to this was developed in the Notes, p165ff



$\bar{u} = 1.3\,m/s$, $Re = 3681.8$ (turbulent)

$u = U_{max}\left(1 - r/R\right)^{1/6}$; $u_{av} = 0.791\,U_{max}$

The heat gained by the oil $= \dot{m} c_p \Delta T = 0.347(2328)(375-12)$

$= 293,237.2$ W

The surface area $= \pi D L = \pi(0.02)(12) = 0.75398\ m^2$

$\therefore$ Required heat flux at wall, $q_w = 388,917.93\ W/m^2$

The energy balance equation for the problem is eq. 6.95 in Notes

$$\frac{1}{u r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial x} \quad ; \quad \alpha = k/\rho c_p$$

For fully developed velocity and temperature profiles,

$$\frac{T_w - T}{T_w - T_{bm}} = \text{const} \quad \text{or} \quad \frac{d}{dx}\left(\frac{T_w - T}{T_w - T_{bm}}\right) = 0$$

Also for const. q_w,

$$\frac{dT_{bm}}{dx} = \frac{2 q_w}{R\, u_{av} \rho c_p} = \text{const.} = 30.24\ \frac{°C}{m}$$

(Also from $\frac{(375-12)\,°C}{12m}$)

~~where~~

$$T_{bm} = \frac{2}{R^2 u_{av}}\int_0^R u T r\, dr$$

Since

$$q_w = -k \frac{dT}{dr}\Big|_R = h(T_w - T_{bm}) = \text{const.}$$

one obtains, for a fully developed temperature profile, that

$$\frac{dT_w}{dx} = \frac{dT_{bm}}{dx} = \frac{dT}{dx}$$

The energy equation simplifies to

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \left[\frac{1}{\alpha}\frac{dT_{bm}}{dx} U_{max}\right]\left(1 - \frac{r}{R}\right)^{1/6} r$$

Integrate

$$r\frac{dT}{dr} = \beta \int r\left(1 - \frac{r}{R}\right)^{1/6} dr + C_1$$

$$\eta \frac{dT}{d\eta} = \beta R^2 \int \eta (1-\eta)^{1/6} d\eta + C_1$$

using Tables of integral

$$= \beta R^2 \left\{ \frac{(1-\eta)^{2\frac{1}{6}}}{2\frac{1}{6}} - \frac{(1-\eta)^{1\frac{1}{6}}}{1\frac{1}{6}} \right\} + C_1$$

$$r\frac{dT}{dr} = \beta R^2 \left\{ \frac{6}{13}(1-\eta)^{13/6} - \frac{6}{7}(1-\eta)^{7/6} \right\} + C_1$$

at $r = 0$, $\frac{dT}{dr} = 0$ (symmetry)

$$0 = \beta R^2 \left\{ \frac{6}{13} - \frac{6}{7} \right\} + C_1$$

$$C_1 = \beta R^2 \left\{ \frac{78 - 42}{91} \right\} = \beta R^2 \frac{36}{91}$$

$$\therefore \quad \eta \frac{dT}{d\eta} = \beta R^2 \left\{ \frac{6}{13}(1-\eta)^{13/6} - \frac{6}{7}(1-\eta)^{7/6} \right\} + \beta R^2 \frac{36}{91}$$

$$\frac{dT}{d\eta} = \beta R^2 \left\{ \frac{36}{91}\frac{1}{\eta} + \frac{6}{13}\frac{(1-\eta)^{13/6}}{\eta} - \frac{6}{7}\frac{(1-\eta)^{7/6}}{\eta} \right\}$$

Integrate

$$T = \beta R^2 \left\{ \frac{36}{91}\ln\eta + \frac{6}{13}\int \frac{(1-\eta)^{13/6}}{\eta}\,d\eta - \frac{6}{7}\int \frac{(1-\eta)^{7/6}}{\eta}\,d\eta \right\}$$

$$+ C_2$$

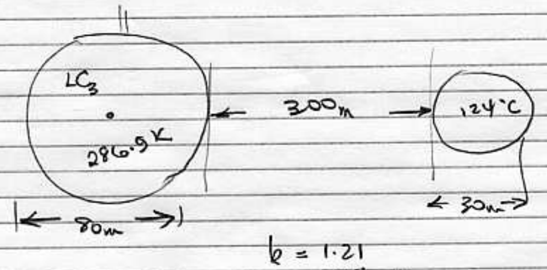
Solve for T_w, T_{bm} and $\left.\frac{dT}{dr}\right|_R$

and substitute into

$$\frac{h}{k} = \frac{-\left.\frac{\partial T}{\partial r}\right|_R}{T_w - T_{bm}}$$

Problem # 4

(a)



$k = 1.21$ W/mK

This is a shape factor problem.

$D = 300 + 40 + 15 = 355$ m

$R_1 = 40$ m

$R_2 = 15$ m

$$S = \frac{4\pi(15)}{\frac{15}{40}\left[1 - \frac{(40/355)^4}{(15/355)^2}\right] - \frac{2(15)}{355}} \quad m$$

$$= 734.482 \text{ m}$$

$$Q = kS\Delta T = \dot{m}\,\Delta H_v$$

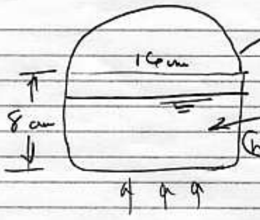
$$\dot{m} = \frac{(1.21)(734.482)(124-13.75)}{(355.9)(10^3)}$$

$$= 0.2753 \text{ kg/s}$$

$$\therefore \dot{V} = \frac{\dot{m}}{\rho} = \frac{0.2753}{15.3} = 0.018 \text{ m}^3/\text{s}$$

→

(b)



$m = 750\,g$

$T_a = 20^\circ C$

1.2 litres or 1.1976 kg water

$h = 36\ W/m^2K$

$Q = 550\,W$

Assume no vapor is lost from the system, i.e. total mass remains constant

Energy balance on system

input + generation = output + accum.

$$Q = hA(T - T_a) + (m_1 C_{P_1} + m_2 C_{P_2})\frac{dT}{dt}$$

if the pot and the water can be assumed to be exactly at the same temperature at all times.

$$\beta \frac{d\theta}{dt} + \gamma\theta = Q \quad ; \quad \begin{cases} \theta = T - T_a \\ \beta = m_1 C_{P_1} + m_2 C_{P_2} \\ \gamma = hA \end{cases}$$

$$\frac{d\theta}{dt} + P\theta = \frac{Q}{\beta} \ ; \quad P = \frac{\gamma}{\beta}$$

$$\theta e^{\int P dt} = \int e^{\int P dt} \frac{Q}{\beta} dt + C$$

$$\theta e^{Pt} = \frac{Q}{\beta}\int e^{Pt} dt + C$$

$$= \frac{Q}{\beta}\frac{e^{Pt}}{P} + C$$

$$\theta = \frac{Q}{\beta}\frac{\beta}{\gamma} + Ce^{-Pt} \quad \text{with} \quad \begin{matrix} t = 0 \\ \theta = \theta_0 \end{matrix}$$

$$\theta = T - T_a = \theta_0 e^{-\frac{\gamma}{\beta}t} + \frac{Q}{\gamma}\left(1 - e^{-\frac{\gamma}{\beta}t}\right)$$

$\longrightarrow$

When the water is placed in the container, the combined system temperature re-adjusts.

Heat gain by water = Heat loss metal

With 1.2 litres of water at 7°C, or 1.1976 kg

$$1.1976(4120)(T_f - 7) = 0.75(465)(20 - T_f)$$

$$T_f = 7.858\,°C$$

The heating is in 2 stages:

- 7.86 °C to 20 °C, pot gains heat from both the flame and ambient air.

- 20 °C to 96 °C, pot loses heat to ambient but gains from the flame.

Stage 1 $\quad \gamma = -hA$ since heat is input by convection as well

$$\beta = 0.75(465) + 1.1976(4120) = 5282.862$$

$$A = \pi DH + 2\pi D^2 = \pi(0.16)(0.08) + 2\pi(0.08)^2 = 0.080425\,m^2$$

$$hA = 2.895$$

$$\theta_0 = 7.857 - 20$$

$$12.1419\, e^{\frac{2.895}{5282.862}t} = \frac{550}{2.895}\left(e^{\frac{2.895}{5282.862}t} - 1\right)$$

$$e^{\frac{2.895}{5282.862}t} = 1.0683$$

$$t = 120.52\ s$$

Stage 2 $\quad r = hA$, since heat is lost by convection

For this stage, $\theta_0 = 0$

Hence

$$T - T_a = 189.9604 \left(1 - e^{-5.4805(10^{-4})t}\right)$$

$$T_a = 20°C, \quad T = 96°C$$

$$0.4 = 1 - e^{-5.4804(10^{-4})t}$$

$$\therefore \quad t = 932.3s$$

Total time taken = $120.52 + 932.3 = 1052.82$ s

or 17 minutes, 32.8 s $\longrightarrow$

(ii) This is the minimum time because it is difficult to prevent loss of vapor without using a pressure vessel. Vaporization of water requires substantial heat input per unit mass. Thus even if a small mass of water is lost, a considerable amount of heat is used to supply the latent heat; yet the sensible heat gain by liquid is not significantly reduced.