The University of Calgary
Department of Philosophy

Philosophy 479 Lec 01
LOGIC III
Winter 2007 — Richard Zach
Course Outline

Instructor:	**Richard Zach**
Office:	1254 Social Sciences
Office Hours:	MW 1:30–2:15 (tentative) or by appointment
Phone:	220–3170 email: rzach@ucalgary.ca
Lectures:	MW 5:00–6:15
	126 Science Theatres

Course Description

This course is a continuation of Phil 379 (Logic II). Specifically, we will focus on two famous theorems of symbolic logic due to Kurt Gödel: The Incompleteness Theorems. The first of these states, roughly, that every formal mathematical theory, provided it is sufficiently expressive and free from contradictions, is incomplete in the sense that there are always statements (in fact, true statements) in the language of the theory which the theory can't prove.

In order to prove the Incompleteness Theorem, we'll need to study the expressive power of formal languages and axiomatic theories—this is an important and exciting area in itself. This investigation will lead us naturally to a topic familiar from Logic II, i.e., computability. In Logic III, however, we'll approach computability not via Turing machines, but via the notion of a recursive function. (We will prove, however, that both notions coincide.)

Prerequisites and Preparation

Logic II (PHIL 379) is a prerequisite for this course.

It can't hurt to review the material from PHIL 379, especially Chapters 1, 2, 9, 10, 12 of the textbook (Chapters 1, 2, 3, 9, 12, 13 of the 3rd edition).

Required Text

Peter Smith, *An Introduction to Gödel's Theorems*, Cambridge University Press, 2007

The book is *not* available in the bookstore (it is not yet published); it is available in electronic form on the course website.

Requirements and Evaluation

The course requirements are: A "diagnostic" homework assignment (5%), 4 homework assignments (60%—15% each), and a final project (30%—25% paper, 5% presentation). You must submit all four assignments and the final project.

The final project will consist in a paper (approx. 2500 words) on a topic of your choice related to the material covered in the course. You will give an oral presentation of your paper in the final week of class.

Class participation counts for the remaining 5% of your grade. If you are shy and do not want to speak in class, 4 substantive, serious posts over the course of the term on the online discussion board will earn you an A for participation. Only posts made before the last day of class count. If all your posts are made within a 7-day period, you will receive a maximum credit of 2 grade points for them.

On each problem on an assignment and on the paper and presentation parts of the final project you will receive a letter grade reflecting the level of mastery of the material shown by the work you submit. According to the *Calendar*, letter grades are defined as follows:

A Excellent—superior performance, showing comprehensive understanding of subject matter. (Your solution to an assigned problem shows that you understand the problem and how to solve it; the solution is complete and rigorously correct, and is reasonably direct and elegant.)

B Good—clearly above average performance with knowledge of subject matter generally complete. (You understand the problem and give a complete solution, although there may be minor gaps in the proof, or the solution is correct but circuitous.)

C Satisfactory—basic understanding of the subject matter. (You understand what the question is asking but your solution contains significant errors or gaps.)

D Minimal pass—marginal performance. (You show some knowledge of what is asked, but you don't come near a solution.)

F Fail—Unsatisfactory performance. (It is not clear that you understand what the question is asking, or your proposed solution goes completely in the wrong direction.)

The correspondence of letter grades with grade points is defined in the *Calendar* (A = 4, B = 3, C = 2, D = 1, F = 0). "Slash" grades are possible with grade point values 0.5 below the higher grade (e.g., A/B = 3.5).

In computing your final grade, your marks will be converted to grade points and averaged according to the weights given above. The final grade will be the letter grade corresponding to the weighted average of your assignments, exams, and participation plus a margin of 0.1. For the final grade, +'s and −'s are possible, too; as defined in the *Calendar*, +/− adds/subtracts 0.3 grade points. In other words, a course average of 3.9 or higher receives an A; between 3.6 and 3.9, an A–; between 3.2 and 3.6, a B+; between 2.9 and 3.2, a B; and so on. There is no D− grade; to earn a D you require a course average of at least 0.9. The A+ grade is reserved for "truly outstanding" performance.

Assignments and Policies

Late work and extensions. Assignments handed in late will be penalized by the equivalent of one grade point per calendar day. If you turn an assignment in late, you must give it to me personally or put it in the department drop-box (it will then be date-stamped by department staff). Note that the drop-boxes are cleared at 4 pm, the department closes at 4:30 pm on weekdays and *is closed Saturdays and Sundays*.

Collaboration. Collaboration on homework assignments is encouraged. However, you must write up your own solutions, and obviously you must not simply copy someone else's solutions. You are also required to list the names of the students with whom you've collaborated on the assignment. **If you collaborate without following these instructions, it constitutes cheating.**

Plagiarism. You might think that it's only plagiarism if you copy a term paper off the Internet. However, you can also plagiarize in a logic course, e.g., by copying a proof verbatim from the textbook (and only making the necessary changes to apply it to the assigned problem.) The point of logic problems which are similar to the proofs in the text is to make you work through those proofs, understand them, and then prove a similar result on the homework. Hence, all homework solutions must be in your own words; copying or paraphrasing closely from the text will be treated as plagiarism and results in a failing grade in the course, and a report to the Dean's office.

Checking your grades and reappraisals of work. University policies for reappraisal of term work and final grades apply (see the *Calendar* section "Reappraisal of Grades and Academic Appeals"). In particular, term work (homework assignments, final paper) will only be reappraised within 15 days of the date you are advised of your marks. Please keep track of your assignments (make sure to pick them up in lecture or in office hours) and your marks (check them on the website) and compare them with the graded work returned to you.

Course Website

A course website on U of C's BlackBoard server has been set up. You will be automatically registered if you're registered in the class. To access the BlackBoard site, you can either go directly to blackboard.ucalgary.ca and log in with your UCIT account name and password, or you can access it through the myUofC portal (my.ucalgary.ca; log in with your eID). If you don't have an eID or UCIT account, see elearn.ucalgary.ca/help.html.

You must log on at least once by the end of the second week of class.

Tentative Syllabus and Due Dates

This is a tentative syllabus to give you a rough idea what parts of the book we will cover when. Due dates are subject to change.

Week 1: Introduction, Review (Jan 8, 10). Chapters 1–3

Learning goals: Understanding enumerability, effective enumerability, decidability; axiomatic theories and their properties.

Week 2: Expressing and Capturing Numerical Properties (Jan 15, 17): Chapters 4–6.

Learning goals: Understanding how properties and relations of numbers can be expressed in the language of arithmetic, and how they can be captured by formal theories. Understanding the connections between expressive power of theories and decidability.

Diagnostic Assignment due Monday, Jan 15 (covers material from Phil 379).

Week 3: Formal Arithmetics (Jan 22, 24): Chapters 8–10

Learning goals: Acquaintance with formal theories of arithmetic, facility with proving things in these theories. Understanding induction.

Assignment 1 due Wednesday, Jan 24

Week 4: Primitive Recursive Functions (Jan 29, 31). Chapters 11–12.

Learning goals: Understanding primitive recursion, facility with defining primitive recursive functions.

Week 5: Capturing Primitive Recursive Functions (Feb 5, 7). Chapter 13

Learning goals: Understanding why all p.r. functions are Σ_1, and why all Σ_1 functions can be captured in Q.

Week 6: Arithmetization of Syntax (Feb 12, 14): Chapter 15.

Learning goals: Understanding Gödel numbering and why syntactic properties are primitive recursive.

Assignment 2 due Wednesday, Feb 14

Week 7: Incompleteness (Feb 26, 28). Chapter 16, 17

Learning goals: Understanding and proving Gödel's First Incompleteness Theorem.

Week 8: Diagonalization (Mar 5, 7): Chapters 19–21

Learning goals: Generalizing the proof of the first incompleteness theorem; understanding and applying the diagonalization lemma to get Tarski's Theorem about the undefinability of truth.

Week 9: Second-order Logic and Arithmetic (Mar 14, **no class Monday, March 12**) Chapter 22

Learning goals: Understanding quantification over sets; expressive power of 2nd order logic; understanding second-order arithmetic.

Assignment 3 due Wednesday, Mar 14

Week 10: Provability Predicates and the Second Incompleteness Theorem (Mar 19, 21): Chapters 24–27.

Learning goals: Understanding formalized consistency statements and provability conditions. The second incompleteness theorem. Reflection principles.

Week 11: Recursive Functions (Mar 26, 28): Chapters 29, 30.

Learning goals: Understanding μ-recursive functions.

Week 12: Recursive Functions and Turing Machines (Apr 2, 4): Chapter 31–34.

Assignment 4 due Wednesday, Apr 4

Learning goals: Understanding why μ-recursive functions are Turing computable and vice versa. The Church-Turing Thesis.

Week 13: Student Presentations (Apr 9, 11).

Final project due Wednesday, April 11