Applying Polyphase Filter in Airborne Strapdown Inertial Gravimetry

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BIOGRAPHY

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ABSTRACT

Gravity information can be obtained by subtracting the kinematic acceleration determined by GPS from the specific force measured by INS. Because the sampling rates of the two systems are usually different, resampling the data from both systems to the same point with a constant sampling rate is an important issue. INS clock errors can cause non-uniform sampling spacing in the INS data. This requires a conversion between non-uniform spacing and uniform spacing.

Because of its computational efficiency and flexibility, the polyphase filter is widely used in the resampling operation of electrical signal and radar signal. Other advantage of using the polyphase filter is the sampling rate conversion between non-uniform spacing and uniform spacing. In this paper the polyphase filter is applied to the resampling process of GPS/INS data.

First the paper describes the principle and implementation of the polyphase filter. Then applying polyphase filter to resample INS data to the GPS data rate is discussed. Test results show the advantages of using polyphase filter in the resampling process of the airborne gravity system.

INTRODUCTION

For a kinematic system with multiple sensors, it is often necessary to resample the kinematic data to the same sample points with constant sample spacing. In many cases the output rates of the sensors are different, and the digital data are not sampled at the same points. Thus, a resampling operation is often employed in the kinematic data process of multi-sensor systems.

In general there are three different cases for the sampling rate conversion. They are:

- Resampling from uniform input spacing to uniform output spacing
- Resampling from non-uniform input spacing to uniform output spacing
- Resampling from uniform input spacing to non-uniform output spacing

In many cases the sampling rates of input data and output data are different. If the digital signal or data is resampled to the lower rate, the low-pass filter is necessary to avoid the aliasing problem.

The use of FIR filter to alter the sampling rate of a uniformly sampled digital signal has been discussed in Crochiere, R.E., and L.R. Rabiner (1981). Because of its flexibility and efficiency, the polyphase filter has been used to resample SAR signal in the SAR image process, see W.G. Carrara et al. (1995). The important advantage of using polyphase filter is that the resampling operation and low-pass filtering are combined into a one-step process by the polyphase filter.

The airborne inertial gravity system developed by Intermap is based on the integration of GPS and INS sub-systems, see M. Wei and K. Tennant (1999). Because the GPS and INS data are captured at different rates, interpolating the INS data to the GPS data points, which have the lower sample rate, is needed. In this paper the polyphase filter is applied to resample the digital data of the inertial system of the airborne gravity system.

SIGNAL RESAMPLING AND POLYPHASE FILETR

Fundamental to the concept of digital signal resampling is the sampling theorem. A band-limited analog signal can be reconstructed from its discrete samples if the sampling rate 1/T is greater than twice the highest frequency (the Nyquist rate) of the analog signal. Based on the sampling theorem the signal at time t can be covered through a convolution with a sinc function

$$x_a(t) = \sum_{k=-\infty}^{\infty} x(k)\mathrm{sinc}\left(\frac{t-kT}{T}\right) \qquad (1)$$

where the sinc function is defined by

$$\mathrm{sinc}(t) = \frac{\sin(pt)}{pt} \qquad (2)$$

A general form of conversion between the sampling rate of the input signal *x(k)* and the output rate of a resampled signal *y(m)* is given by

$$y(m) = \sum_{k=-\infty}^{\infty} x(k)h(mM - kL) \qquad (3)$$

where L is the upsampling scale and M is the downsampling scale, *h(n)* is the impulse response function of a low-pass filter with a sample spacing of T/L, see Croochiere and Rabiner (1981), Carrara et al. (1995).

Equation (3) implies that the resampling procedure combines two different procedures: upsampling the input signal *x(k)* by the scale L and downsampling the resulting signal sequence by the factor M. In order to meet the Nyquist sampling criterion of the sampling theorem, it is necessary to apply a low-pass filter *h(n)* to the input signal.

Equation (3) represents an ideal case of the resampling procedure with infinite signal sequence. In practice, the input signal is only available over a limited period. Thus a digital filter approximates the ideal filter to reduce the summation in (3) to a finite number of terms.

In general there are two ways to implement the filtering process of a polyphase filter: input-centered convolution and output-centered convolution. In the input-centered convolution, the center of the impulse response function is located at the input sample and the contribution of input signal on the filtering convolution will be calculated; details of calculation of the input centered convolution see Carrara et al. (1995).

In the output-centered convolution, the center of impulse response function *h(n)* is located at the center of each output sample and a convolution with the input signal is calculated over the filtering length. Figure 1 shows the principle of the output-centered convolution of polyphase filter.

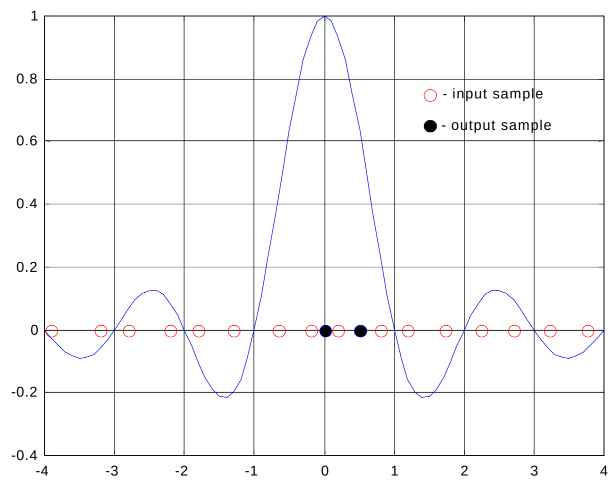

Figure 1 Output centered convolution

As discussed above, the advantage of using a polyphase filter is that the resampling operation, or interpolation process, and the low-pass filtering can be combined into a single step. In Figure 2, the polyphase filter process is compared to the interpolation method used in the resampling process.

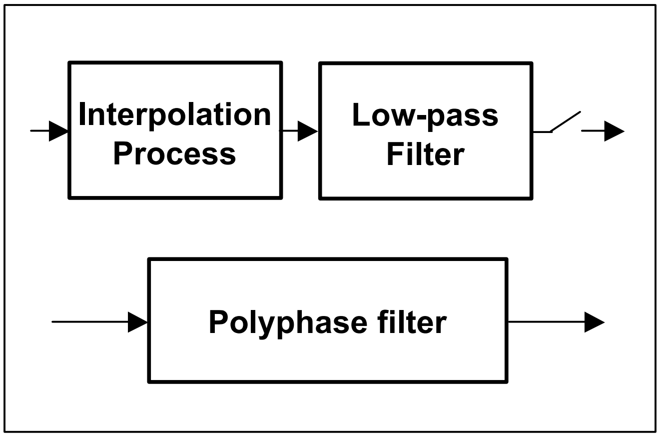

Figure 2 Comparison of resampling procedures

The polyphase filter is also very flexible. The sampling spacing of the input data and output data can be uniform or non-uniform. It can convert the digital data between uniform spacing and non-uniform spacing. The sampling rate of the input data and output data can also be different.

APPLICATION IN AIRBORNE GRAVITY SYSTEM

Intermap has developed a new airborne gravity system (AGS) based on the strapdown inertial system to determine the airborne gravity disturbance. The components include a strapdown inertial system (a Honeywell H-770 or LASERREF III system, that performs at 0.8 nm/hr) and two GPS receivers. The measurement output rate of the H-770 system has been changed to 1200 Hz while the output rate of LASEREF III is at 50 Hz. Both the base and airborne stations for DGPS use Ashtech Z-12 GPS receivers. The AGS synchronizes INS recording times with GPS times and records both INS data and airborne GPS data.

Determining the airborne gravity anomaly or disturbance is done in two steps. First, the attitude, position and velocity of the airborne gravity system are computed using the DGPS/INS integration software. In the second step the airborne gravity anomaly or disturbance is computed by the combination of accelerations measured by the strapdown inertial system and the GPS derived acceleration. To reduce the effect of measurement noise, low-pass filtering is applied to DGPS and INS results.

The polyphase filter is applied to resample the inertial acceleration output to the GPS data points. The process flowchart of the airborne gravity systems including the resampling procedure is shown in Figure 3.

The airborne gravity disturbance results contain long-term errors due to INS measurement biases and drifts. To eliminate them, a least-squares adjustment is applied to the differences of the airborne gravity disturbance at crossover points.

To reduce the effect of sensor noise on the gravity disturbance in the high frequency bandwidth, low-pass filtering is applied to the airborne gravity disturbance measurements. For most geodetic applications, e.g. geoid determination, a low-pass filter with cutoff frequency of either 0.011 Hz or 0.083 Hz, corresponding to the 1/e response at 90 seconds and 120 seconds, is often used. The detailed description of the process of the airborne inertial gravity system is described in M. Wei and K. Tennant (1999).

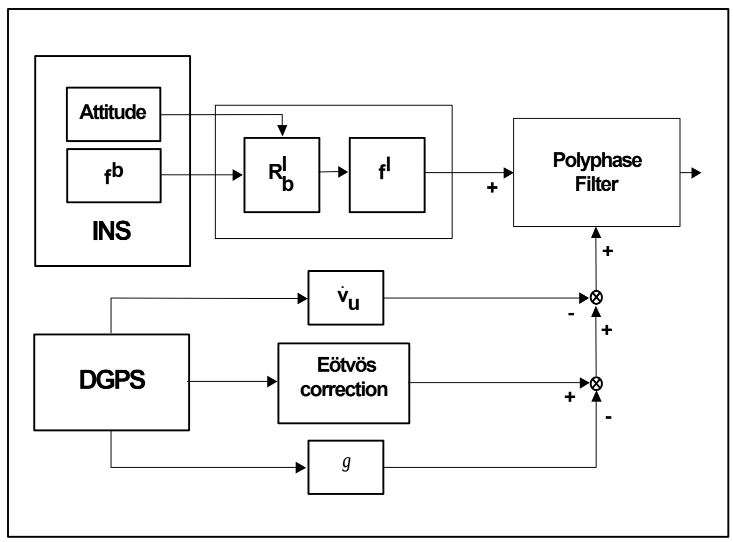

Figure 3 Airborne inertial gravity system with resampling process

TEST RESULTS

In the spring of 2000, an airborne gravity campaign was carried out to compare the three currently available airborne gravity concepts on a single aircraft; a strapdown inertial navigation system (SINS), a 3-axis inertially stabilized platform and an air-sea gravimeter, see Bruton et al. (2001) for more details.

Figure 4 shows the flight lines of the airborne test. There are six parallel flight lines of 100 km. The spacing between flight lines is about 10 km. In order to establish the crossover points to estimate the bias and drift of the airborne gravity measurements of each line, there are three additional flight lines in the cross-track direction.

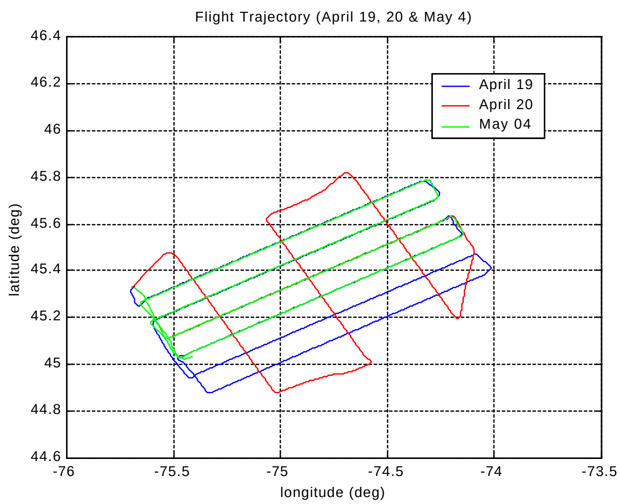

Figure 4 Trajectory of airborne gravity test

Intermap has joined the airborne flight test using the LASEREF III inertial system. The acceleration of LASEREF III is 50 Hz and the GPS data rate is 1 Hz. The polyphase filter was employed to resample the acceleration data of LASEREF III to the GPS data rate. The airborne gravity disturbance measurements are determined by taking the difference of the INS acceleration and the GPS-derived kinematic acceleration. For this test, the low-pass filter with cutoff frequency of 0.083 Hz, corresponding to the 1/e response 120 seconds, was applied to airborne gravity disturbance measurements.

In order compare different resampling procedures, the polyphase filter, linear interpolation and cubic spline interpolation were used in the airborne gravity process.

Table 1 gives the RMS of the difference of the airborne gravity disturbance at crossover points, obtained using different interpolation methods.

No of flights	Polyphase filter	Linear interpol.	Spline interpol.
33	1.94	1.91	2.00

Table 1 RMS of gravity disturbance difference at crossover points (mGal)

To assess the accuracy of the airborne gravity disturbance measurements, the upward continued ground gravity data was converted to the gravity disturbance and compared to the airborne gravity disturbance at flight level. The accuracy of ground gravity anomaly is about 1 mGal and the average spacing of the ground gravity anomaly is about 3 km. The standard deviations of the differences between the measured airborne gravity disturbance and the upward continued gravity disturbance are about 1 - 2 mGal (1 σ) and are almost the same for the different interpolation methods, as shown in Table 2.

Flight line No.	Polyphase filter	Linear interpol.	Spline interpol.
Line 1	2.25	2.27	2.22
Line 2	1.39	1.44	1.45
Line 3	1.20	1.15	1.21
Line 4	1.04	1.02	1.02

Table 2 Standard deviation of comparison of gravity disturbance (mGal)

CONCLUSIONS

The polyphase filter has been successfully used in the airborne inertial gravity system to resample the accelerations of the inertial system. As shown here, this filter can also be a powerful tool to resample kinematic data of multi-sensor systems because it is flexible and efficient. It combines the resampling (or interpolation) process and the low-pass filtering. As well, the implementation of polyphase filter is similar to the FIR filter and therefore easy.

ACKNOWLEDGMENTS

The authors would like to thank Tim Coyne for useful discussion in polyphase filter technique.

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